

Secondary Course

211 - Mathematics

Practical Manual

Course Coordinator
Neeraj Pratap Singh



NATIONAL INSTITUTE OF OPEN SCHOOLING
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Chairman's Message

Dear learner,

As the needs of the society in general, and some groups in particular, keep on changing with time, the methods and techniques required for fulfilling those aspirations also have to be modified accordingly. Education is an instrument of change. The right type of education at right time can bring about positivity in the outlook of society, attitudinal changes to face the new/fresh challenges and the courage to face difficult situations. This can be very effectively achieved by regular periodic curriculum renewal. A static curriculum does not serve any purpose, as it does not cater to the current needs and aspirations of the individual and society.

For this purpose only, educationists from all over the country come together at regular intervals to deliberate on the issues of changes needed and required. As an outcome of such deliberations, the National Curriculum Framework (NCF 2005) came out, which spells out in detail the type of education desirable/needed at various levels of education – primary, elementary, secondary or senior secondary.

Keeping this framework and other national and societal concerns in mind, we have currently revised the curricula of all the subjects at the secondary level, making them current and need based. Textual material production is an integral and essential part of all NIOS programmes offered through open and distance learning system. Therefore, we have taken special care to make the learning material user friendly, interesting and attractive for you.

I would like to thank all the eminent persons involved in making this material interesting and relevant to your needs. I hope you find it appealing and absorbing.

On behalf of National Institute of Open Schooling, I wish you all a bright and successful future.

(Dr. S. S. Jena)

Chairman, NIOS

A Note From the Director

Dear Learner,

The Academic Department at the National Institute of Open Schooling tries to bring you new programmes every now and then in accordance with your needs and requirements. We are now revising the curriculum in all the subjects at the secondary level. In order to bring to you a curriculum which is at par with other boards in the country, we consulted the curriculum in different subjects at the CBSE and Several State Boards of Secondary Education. The National Curriculum Framework developed by the National Council for Educational Research and Training was kept as a reference point. After making a comprehensive comparative study, we developed the curriculum that was functional, related to life situations and simple. Leading educationists of the country were involved and under their guidance, we have been able to revise and update the curriculum.

At the same time, we also had a look at the learning material. We have removed old, outdated information and added new, relevant things and have tried to make it attractive and appealing for you.

I hope you will find the new material that is now in your hands interesting and exciting. Any suggestions for further improvement are welcome.

Let me wish you all a happy and successful future.

(Dr. Kuldeep Agarwal)

Director (Academic), NIOS

A Word With You

Dear Learner,

You might be enjoying Mathematics Book I and Book II provided to you by the National Institute of Open Schooling. Some of the concepts in Mathematics are of abstract nature and in learning such concepts becomes easier when learnt through activities performed in mathematics laboratory. Mathematics activities can be carried out by facilitator and learners to explore, to learn and to create interest of learners in the subject and develop positive attitude towards the subject.

Keeping the above in view, National Institute of Open Schooling has developed a laboratory manual, which is in your hands. This is in addition to the two books of your theory part of mathematics.

In the beginning, this Laboratory Manual has few pages under introduction, given an idea about the importance and meaning of practical work in mathematics.

There are 30 activities given in this manual. Each activity, has detailed instructions about how to perform the experiment and how to take observation to reach at the conclusion.

Though the manual has the scope of recording your observations in tabular form, you are required to maintain a record book, as it carries weightage in practical examination.

In case of any doubt or problem while doing the activity, do not hesitate to write to us.

We hope, you will enjoy performing these activities.

Wishing you all the success.

Yours,

(Neeraj Pratap Singh)
Senior Executive Officer (Maths)

Introduction

It is a general saying that mathematics is by doing only. The concepts for which the proof/verification is done by experimentation or activities are better understood by learners and are retained in their brains for longer period of time. Jen Piaget, an psychologist, while writing his thesis on concept formation in children, has brought out that all abstract concepts, can be brought down to the concrete operational stage, and can be understood and retained in a better way. For example, if the abstract concept of number two, is illustrated by showing two apples, two oranges or any other two similar objects, which the learners can touch and handle, the learner understands in better way.

The human brain is capable of storing only limited amount of information. The information (concepts) which are repeatedly learnt and practiced, are permanently stored in the brains of children which help in learning of concepts. The activities which are repeatedly done for understanding concepts help the understanding for those concepts.

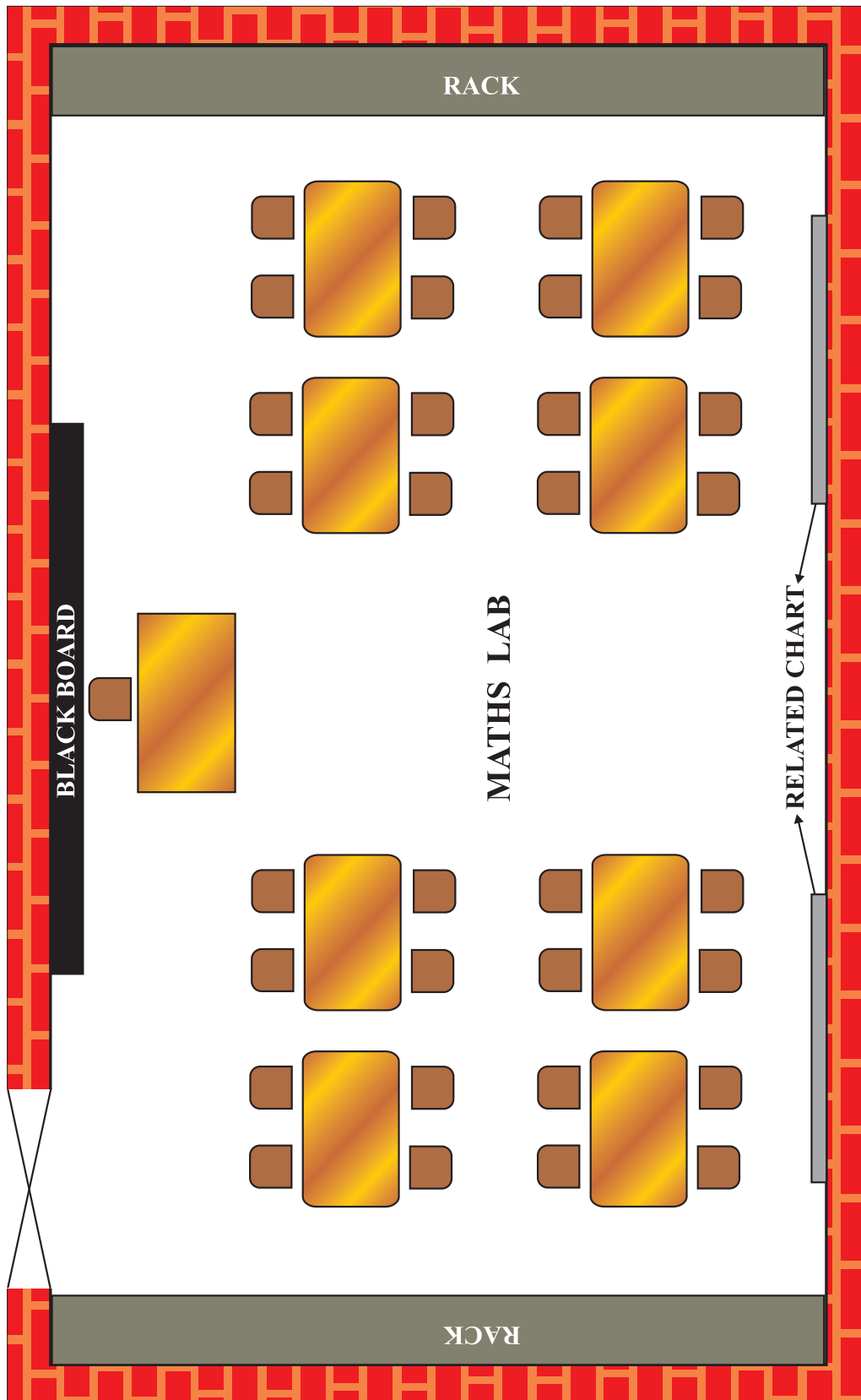
Mathematics laboratory is a place where learners can learn and explore difficult mathematical concepts and verify mathematical facts, formulae and theorems/results through a variety of activities and handling related projects using non-costly materials available in their environment. A mathematics laboratory can create mathematical awareness, skill building, positive attitudes towards the subject and above all ideas of learning by doing.

It is the place where learners can learn certain concepts using concrete objects and verify mathematical facts. Formulae and properties by using modes, measurements and the other activities. Here the learners handle the concrete materials, make models of their own by their own imagination and verify the facts/formulae.

Design and Lay out of the Mathematics Laboratory

In mathematics laboratory there should be sufficient space to accommodate atleast 30 learners for doing activities/experiments at a time.

The rough design is given here:



Materials Required: Sheets of paper of different Colours, Glazed paper scales, Wooden boards, Nails, Threads, Thermo cole piece, Cardboard square and Triangular grids, Pins and Clips, Wooden and paper strip, Cutter (paper), Scissor, Adhesive/fevicol, sketch pen, Gun geometry box (Bigger – Wooden), Graph paper (inches/cm both, Pencils of different colours, Colour box, Knobs, Tracing paper

Human Resource: It is desirable to have a laboratory assistant (with mathematics background), in charge of the mathematical lab. He expected to have special skills required to handle different instruments, needed for practical work. He should be able to repair things, if they are not in order and keep the materials ready for carrying out activities in the following days.

Time – Desired: 15% to 20% of total time for mathematics syllabus to be devoted to mathematics laboratory.

Scheme of Evaluation: 15 marks

The division of marks in the examination can be done as follows:

Activity	Mark
Assessment of Activity Performed/Records of activities prepared	10
Viva – voce	5
Total	15

- i) The proposed practical test is suggested to be held at least 15 days before the theory examination.
- ii) Every students may be given two activities out of which he has to select one and perform it these (in case, the students not comfortable with the given activities, he may be allowed to select one activity of his choice)
- iii) Viva-voce can be done at the examination centre by asking questions related to the activity/project he/she has done.

Contents

S.No.	List of Activities	Pages
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5.	To find the HCF of two given numbers by division method.	9
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ACTIVITY 1



Notes

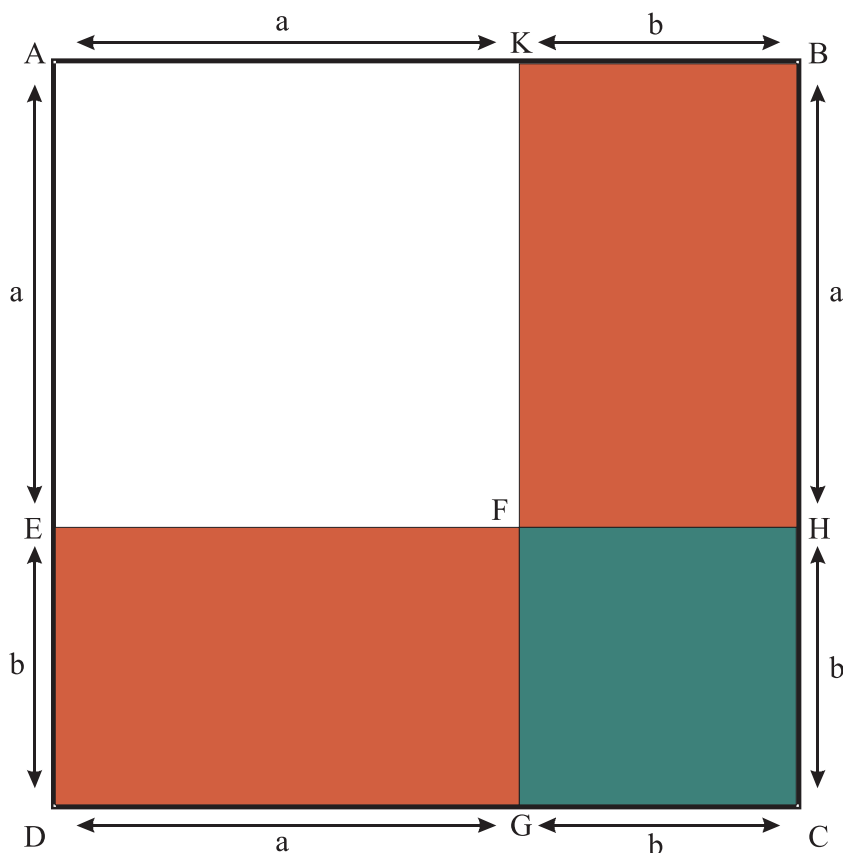
Title: Verification of the identity $(a+b)^2 = a^2 + 2ab + b^2$

Expected background knowledge:- Area of a square and a rectangle.

Objectives: After performing this activity the learner will be able to verify and demonstrate the identity $(a+b)^2 = a^2 + 2ab + b^2$

Materials required:

- (i) Cardboard
- (ii) White chart paper
- (iii) Two glazed papers of different colours, say red and green
- (iv) Pair of scissors
- (v) Gum
- (vi) Coloured ball point pens.
- (vii) Pencil and geometrical instruments.





Notes

Preparation for the activity:

- (i) On a white chart paper, draw a square ABCD of side $(a+b)$ units (say $a=7\text{cm}$, $b=4\text{cm}$) Cut it out and paste it on the cardboard.
- (ii) Cut off two rectangles of dimensions $a \times b$ ($7\text{cm} \times 4\text{cm}$) from red colour paper and a square of side b (4cm) from green colour paper.
- (iii) Paste these cutouts on the square ABCD as shown in the figure and name them as rectangle EFGD, square FHCG and rectangle KBHF

Demonstration and Use

In the figure, area of square ABCD $= (AB)^2 = (a+b)^2 \text{ unit}^2$

Area of square AKFE $= (AK)^2 = a^2 \text{ unit}^2$.

Area of rectangle KBHF $= (KF \times FH) = a \times b = ab \text{ unit}^2$

Area of square FHCG $= (HC)^2 = b^2 \text{ unit}^2$

Area of rectangle EFGD $= ED \times GD = a \times b = ab \text{ unit}^2$

From the figure we may observe that:

Area of square ABCD = Area of square AKFE + area of rectangle KBHF + area of square FHCG + area of rectangle EFGD.

$$\begin{aligned} \text{i.e. } (a + b)^2 &= a^2 + ab + b^2 + ab \\ &= a^2 + 2ab + b^2 \end{aligned}$$

Conclusion:

$$(a + b)^2 = a^2 + 2ab + b^2$$

ACTIVITY 2



Notes

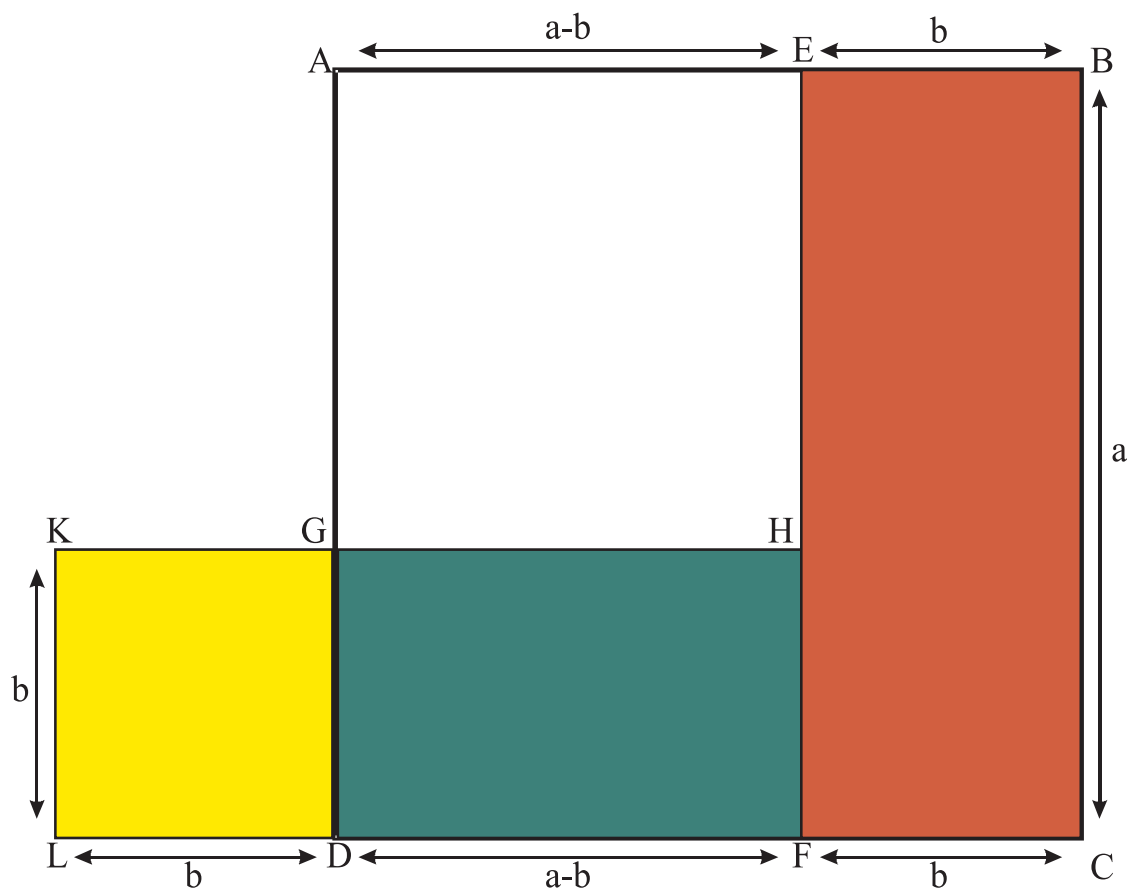
Title: Verification of the identity $(a - b)^2 = a^2 - 2ab + b^2$

Expected background knowledge: Area of a square and a rectangle.

Objectives: After performing this activity, the learner will be able to verify and demonstrate the identity $(a - b)^2 = a^2 - 2ab + b^2$.

Materials required:

- (i) Cardboard
- (ii) White chart paper
- (iii) Three glazed papers of different colours, say red green and yellow.
- (iv) Pair of scissors
- (v) Gum
- (vi) Coloured ball point pens.
- (vii) Pencil and geometrical instruments.





Notes

Preparation for the activity:

- (i) On a white chart paper draw a square ABCD of side a [say $a = 10\text{cm}$], cut it out and paste it on the cardboard.
- (ii) Cut off a rectangle of dimensions $a \times b$ (say $a = 10\text{cm}$, $b = 4\text{ cm}$) from red colour paper, a rectangle of dimensions $(a - b) \times b$ from green colour paper (Here $a - b = 6\text{ cm}$ and $b = 4\text{ cm}$) and a square of side b ($b = 4\text{cm}$) from yellow colour paper.
- (iii) Paste these cut outs on the square ABCD as shown in the figure and name them as rectangle EBCF, rectangle GHFD and square KGDL.

Demonstration and Use

$$\text{Area of square ABCD} = (BC)^2 = a^2 \text{ unit}^2$$

$$\text{Area of square AEHG} = (AE)^2 = (a - b)^2 \text{ unit}^2.$$

$$\text{Area of rectangle EBCF} = (BC \times EB) = ab \text{ unit}^2$$

$$\text{Area of rectangle GHFD} = (GH \times HF) = (a - b)b \text{ unit}^2$$

$$\text{Area of square KGDL} = (KL)^2 = b^2 \text{ unit}^2$$

$$\text{Area of rectangle KHFL} = (KH \times HF) = ab \text{ unit}^2$$

From the figure we may observe that:

$$\text{Area of square AEHG} = \text{Area of square ABCD} +$$

$$\text{Area of square KGDL} - \text{Area of rectangle EBCF} -$$

$$\text{Area of rectangle KHFL}.$$

$$\begin{aligned} \text{i.e. } (a - b)^2 &= a^2 + b^2 - ab - ab \\ &= a^2 - 2ab + b^2 \end{aligned}$$

Conclusion:

$$(a - b)^2 = a^2 - 2ab + b^2$$

ACTIVITY 3

Title: Verification of the Identity: $a^2 - b^2 = (a + b)(a - b)$

Expected background knowledge: Areas of quadrilaterals.

Objectives: After performing the activity, the learner will be able to verify and demonstrate the identity: $a^2 - b^2 = (a + b)(a - b)$

Materials required:

- (i) Cardboard.
- (ii) Glazed paper of different colours.
- (iii) Pair of scissors.
- (iv) Pencil and geometrical instruments.
- (v) Gum.
- (vi) Sketch pens.

Preparation for the Activity :

- (i) Take a cardboard sheet
- (ii) Paste on it a square (of side a) made on blue paper. Area of this square is a^2
- (iii) Make another square of side b ($b < a$) of area b^2 on yellow paper.

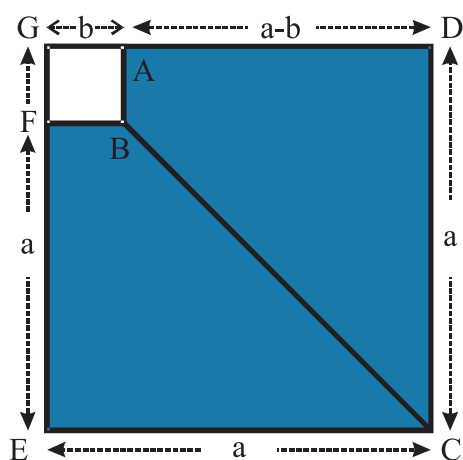


Fig. (i)

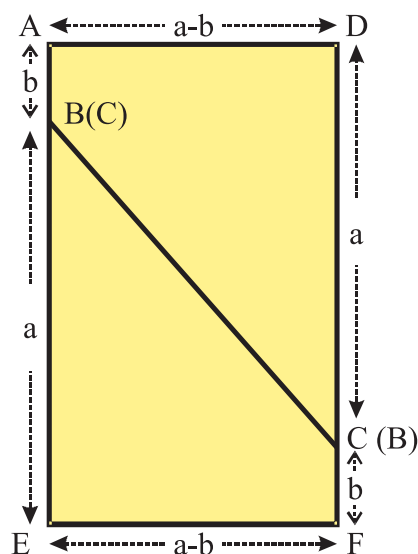


Fig. (ii)

Notes





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- (iv) Paste this smaller square (of side b) on the one side of bigger square [as shown in Fig. (i)]
- (v) Cut the remaining portion ADCEFB and cut it along BC and join them as shown in Fig. (ii) above

Demonstration and Use

- (i) Area of region ADCEFB in Fig. (i) $= a^2 - b^2$
- (ii) The breadth of rectangle in Fig (ii) is $a + b$ and length is $(a - b)$

Its area $= (a + b)(a - b)$

As the region ADCEFB has been transformed into Fig. (ii)

$$a^2 - b^2 = (a + b)(a - b)$$

Conclusion: $a^2 - b^2 = (a + b)(a - b)$

ACTIVITY 4



Notes

Title: Verification of the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Expected background knowledge:

- (i) Knowledge about vertices, edges and faces of cubes and cuboids.
- (ii) Volume and surface area of a cube and a cuboid

Objectives: After performing this activity, the learner will be able to verify and demonstrate the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Materials required:

- (i) Acrylic Sheets
- (ii) Wooden Board
- (iii) Sketch Pens
- (iv) Glazed papers
- (v) Fevicol
- (vi) Pair of Scissors
- (vii) Geometry Box

Preparation for the activity:

Take $a = 3$ cm (say) and $b = 1$ cm making $a + b = 4$ cm.

- (i) Make a cube of side 3 cm from a wooden board
- (ii) Make another cube of side 1 cm from the wooden board
- (iii) Make three cuboids of dimensions 3 cm x 3cm x 1 cm and another three of dimensions 3 cm x 1 cm x 1 cm from the wooden board.
- (iv) Using acrylic sheet, make a cube of side 4 cm.

Demonstration and Use

- (i) The cube of side 4 cm represents $(a + b)^3$ (Fig.5)
- (ii) The cube of side 3 cm represents a^3 (Fig.1)
- (iii) The cube of side 1 cm represents b^3 (Fig.4)
- (iv) A cuboid of 3 cm x 3 cm x 1 cm represents $a^2 b$ (Fig.2)
Thus three such cuboids = $3 a^2 b$
- (v) Similarly a cuboid of 3 cm x 1 cm x 1 cm = ab^2 (Fig.3)
Thus three such cuboids = $3 ab^2$



Notes

Place all these cubes and cuboids in the acrylic cube in such a way that these all fill in the acrylic cube completely showing thereby that the cube of volume $(a+b)^3$ is equal to the sum of volumes of cuboids & cubes of volumes a^3 , b^3 , $3 a^2b$ and $3 ab^2$

Conclusion

$$(a+b)^3 = a^3 + 3 a^2b + 3 ab^2 + b^3$$

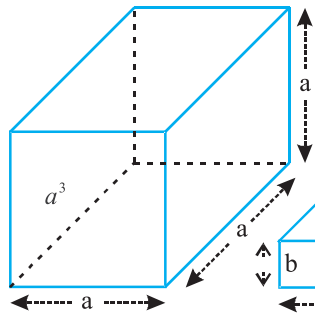


Fig. (1)

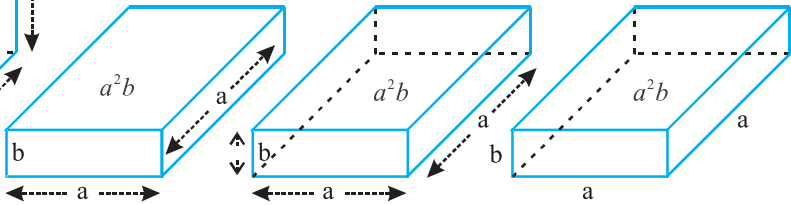


Fig. (2)

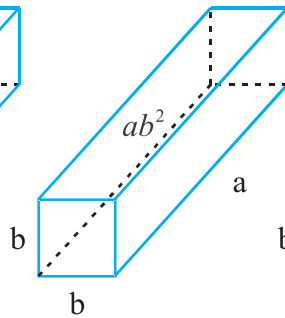
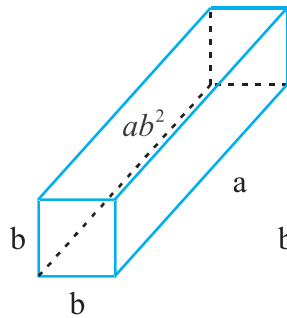


Fig. (3)

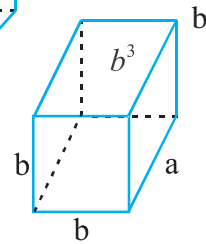
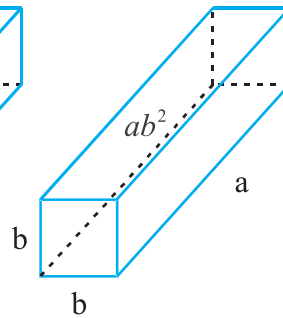


Fig. (4)

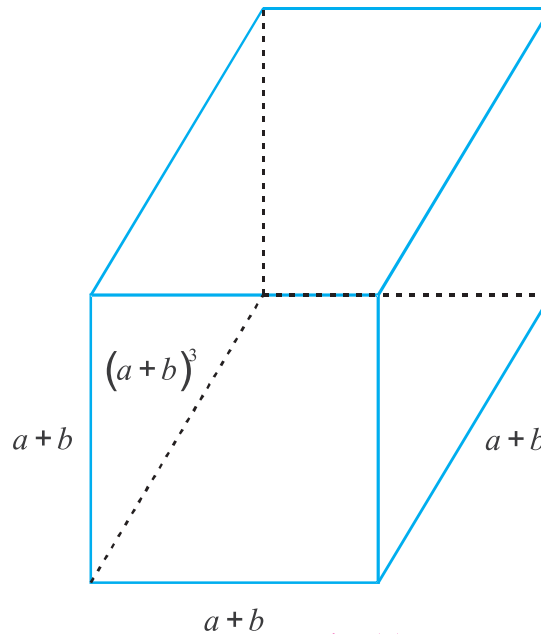


Fig. (5)

ACTIVITY 5

Title: To find the HCF of two given numbers by division method.

Expected background knowledge:

- (i) Factors of numbers
- (ii) Division of numbers

Objectives:

- (i) After completing this activity, the learner will be able to find the HCF of any two given numbers
- (ii) He will be able to find the largest number by which the two numbers can be divided.

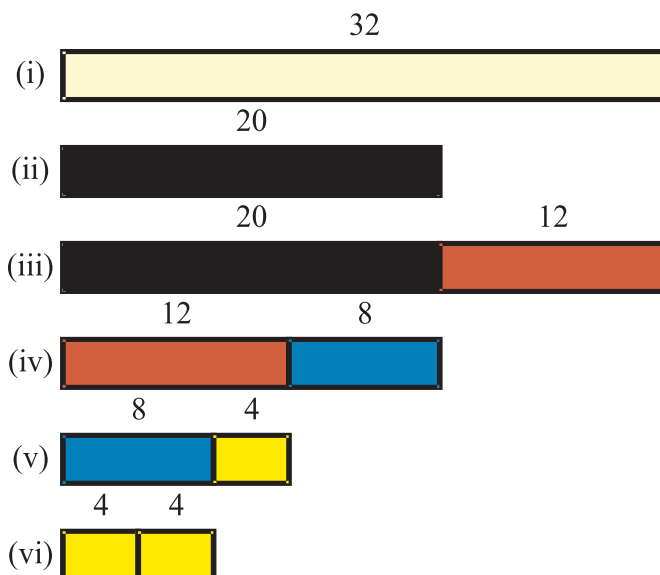
Materials required :

- (i) Cardboard – 5 strips of 2 cm width
- (ii) Sketch Pens
- (iii) Pair of Scissors
- (iv) Fevicol
- (v) Scale
- (vi) Pencil and eraser

Preparation for the activity:

Suppose we have to find the HCF of 20 and 32. Carry out the following steps:

- (i) Cut 2 cm wide cardboard strips to get two pieces of length 32 cm each, two pieces of length 20 cm each, two pieces of length 12 cm each, two pieces of length 8 cm each and three pieces of length 4 cm each.
- (ii) Paste the cardboard strips as shown in figure below



Notes





Notes

Demonstration and Use:

The first two strips represent numbers 32 and 20. Finding HCF means finding highest common factor of 32 and 20 i.e. finding that largest length of strip which measures the lengths 32cm and 20 cm exactly.

- a) From the first two strips it is clear that this length can not be 20 cm as 20 cm does not divide 32.
- b) If we keep the strip (ii) along (i), we see that the length of strip of length 12 cm is left out in (i)

$$\begin{array}{r} 20 \overline{) 32} 1 \\ \underline{20} \\ 12 \end{array}$$

- c) From strip (iii) we see that the desired length can not be 12 cm, as it does not divide 20 cm strip exact number of times. From (iv) we see that 12 cm strip covers 20 cm strip once and then 8 cm strip is left over.

$$\begin{array}{r} 12 \overline{) 20} 1 \\ \underline{12} \\ 8 \end{array}$$

- d) From strip (v) we see that desired length can not be 8, as it does not divide 12 cm strip exact number of times. We see that 8 cm strip covers 12 cm strip once and then 4 cm strip is left over.

$$\begin{array}{r} 8 \overline{) 12} 1 \\ \underline{8} \\ 4 \end{array}$$

- e) The strip (vi) of 4 cm divides 8 cm strip exactly.
- Thus we see that 4 cm long strip measures 32 cm and 20 cm strips exactly.
- Thus, 4 is the HCF of 32 and 20

Conclusion:

To find the HCF of two numbers, we have to find the largest number which can divide both given numbers.

ACTIVITY 6



Notes

Title: Equivalent Fractions**Expected background knowledge:** Concept of fractions**Objectives:** After performing this activity, the learner will understand the concept of equivalent fractions.**Materials required:**

- (i) Glazed paper (red)
- (ii) White square sheet
- (iii) String
- (iv) Sketch pens
- (v) Pencil, eraser and Fevicol.

Preparation for the activity:

- a) Mark 6 strips of same size on a square sheet marked S-1, S2, S3, S4, S5 and S6, each strips starting from an initial point representing zero (Fig. ii)
- b) The first strip S1 has 12 squares and represents 1.
- c) The second strip S2 has 2 equal parts of 6 squares each, each part representing $\frac{1}{2}$ (half strip). Thus OA represents $\frac{1}{2}$.
- d) The third strip S3 has 3 equal parts, each of 4 squares. And each part representing $\frac{1}{3}$ (one third of a strip). Thus OB represents $\frac{1}{3}$ and OC represents $\frac{2}{3}$.
- e) The fourth strip S4 has 4 equal parts, each of 3 squares, each part representing $\frac{1}{4}$ (one fourth of a strip). Thus OD, OE and OF on S4 represent $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ respectively.
- f) The fifth strip S5 has 6 equal parts, each of 2 squares, each part representing $\frac{1}{6}$ (one sixth of a strip). Thus OG, OH, OI, OJ and OK represent $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$ and $\frac{5}{6}$ respectively.
- g) The sixth strip S6 has 12 equal parts, each of 1 square, each part representing $\frac{1}{12}$ (One twelfth of a strip) Thus OL, OM, ON, OP, OQ, OR, OS, OT, OU, OV and OW represent $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{6}{12}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$ and $\frac{11}{12}$ respectively.

Demonstration and Use: Using a glazed paper and a thread, equivalent fractions can be shown in the same vertical line as shown in Fig. (i) and Fig. (ii)Thus $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{6}{12}$ and so onHence $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{6}{12}$ are equivalent fractionsSimilarly $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$ etc. are equivalent fractionsSimilarly $\frac{2}{3}$, $\frac{4}{6}$, $\frac{8}{12}$ are equivalent fractions.



Notes

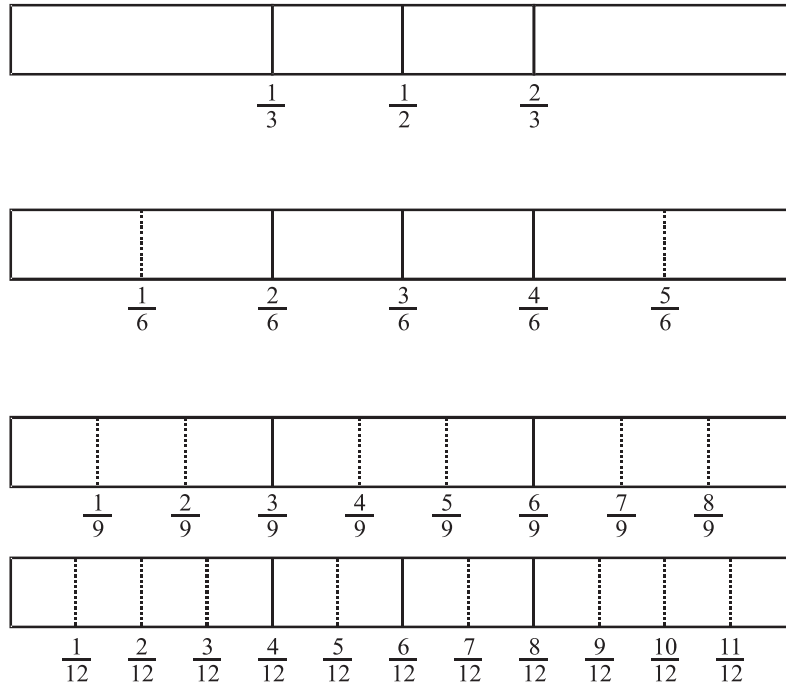


Fig. (i)

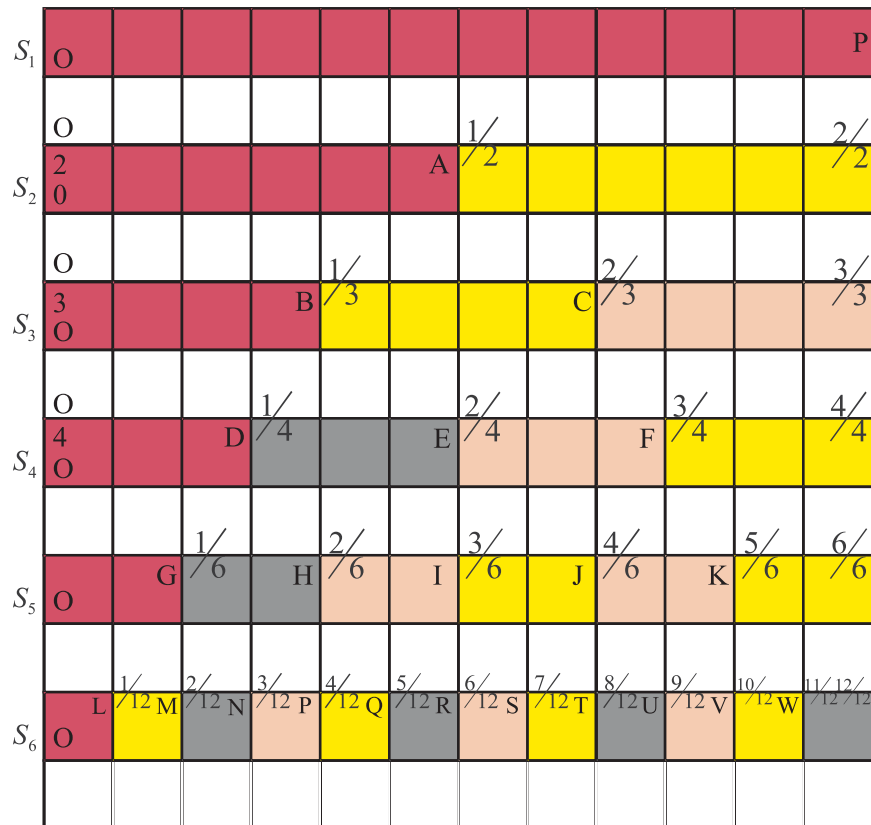


Fig. (ii)

ACTIVITY 7



Notes

Title: To verify that a linear equation in two variables has infinite number of solutions.

Expected background knowledge: Meaning of solution, plotting points on graph paper, reading the coordinates of the points lying on a line drawn on the graph paper.

Objectives: After performing this activity, the learner will be able to demonstrate that a linear equation in two variables has infinite number of solutions.

Materials required: (i) Glazed paper (red)
(ii) Scale
(iii) Pencil and geometrical instruments, (iii) Graph paper

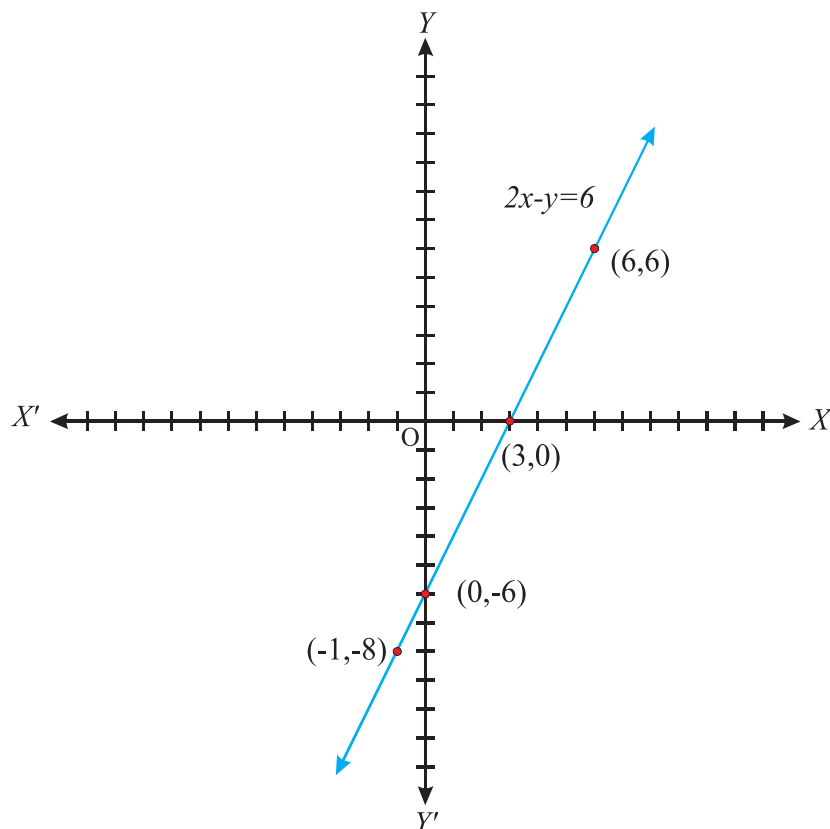
Preparation for the activity:- Consider a linear equation in two variables of the form $ax + by = c$. e.g $2x - y = 6$

Obtain a table of ordered pairs (x, y) which satisfy the given equation

e.g.

x	0	3	-1
y	-6	0	-8

Draw the graph of the given equation on a graph paper as shown in the figure given below.





Notes

Demonstration and Use:

Take any other three points A (6,6) B (1,-4) and C (-4, -14) on the line drawn. Substitute the co-ordinates of these points in the given equation.

i.e. For A (6,6) , $2 \times 6 - 6 = 6 \Rightarrow 6 = 6$

For B (1,-4) , $2 \times 1 - (-4) = 6 \Rightarrow 6 = 6$

For C (-4, -14) , $2 \times -4 - (-14) = 6 \Rightarrow 6 = 6$

Conclusion: You may observe that for each of the three points A,B and C, the given equation is satisfied i.e. LHS of the equation becomes equal to RH.S, therefore coordinates of each of the three points give solution of the given equation. The learner may further note that there are infinite number of such points on the graph of the equation. Hence, a linear equation in two variables has infinite number of solutions.

ACTIVITY 8



Notes

Title: To find the condition for consistency of a system of linear equations in two variables

Expected background knowledge: Plotting points on the graph paper

Objectives: After performing this activity, the learner will be able to find and demonstrate the conditions for a system of linear equations to have a unique solution, infinite solutions or no solution.

Materials required:

- (i) Graph paper
- (ii) Scale
- (iii) Pencil, geometrical instruments.

Preparation for the activity:

Take three pairs of linear equations in two variables of the form

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

e.g. $x + y = 4$ $2x + 3y = 6$ $2x + 3y = 6$
 $2x + 3y = 6$, $4x + 6y = 12$ $4x + 6y = 24$

Consider the first pair of linear equations and obtain a table of ordered pairs (X, Y) for each of the two equations.

e.g. For $x + y = 4$

x	4	6	0
y	0	-2	4

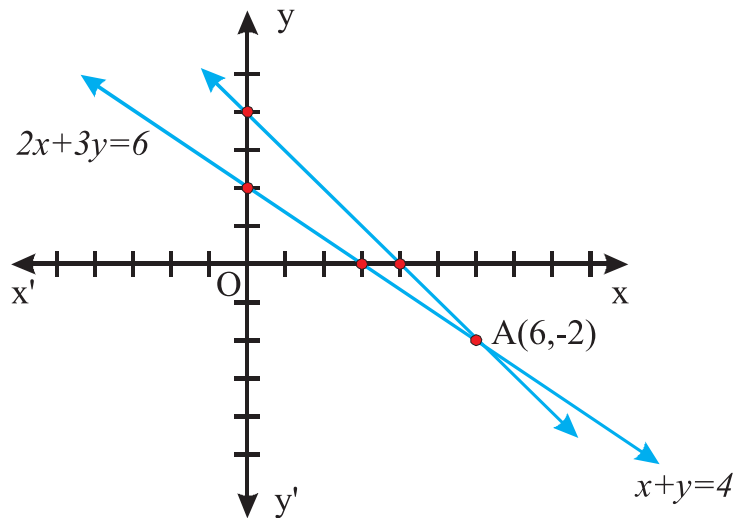
For $2x + 3y = 6$

x	0	3	6
y	2	0	-2



Notes

Draw the graphs of the two equations on the graph paper as shown in the figure given below:



The learners may notice that the two lines representing the equations intersect at the point $A(6, -2)$ and so the two equations have a unique solution, namely

$$x = 6, y = -2.$$

Consider the second pair of linear equations and obtain a table of ordered pair (x, y) for each of the two equations.

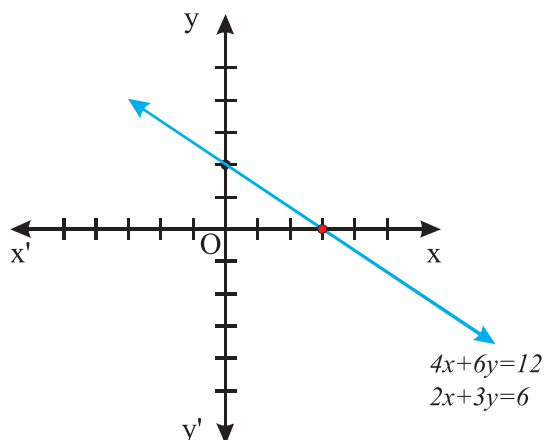
e.g. For $2x + 3y = 6$

x	0	3	6
y	2	0	-2

For $4x + 6y = 12$

x	0	3	6
y	2	0	-2

Draw the graph of two equations on a graph paper as shown in the figure given below:



The learner may notice that the two lines representing the two equations are the same (coincident). The lines have many points (infinite in number) common and so the equations have an infinite number of solutions.

Consider the third pair of linear equations and obtain a table of ordered pairs (x,y) for each of the two equations.

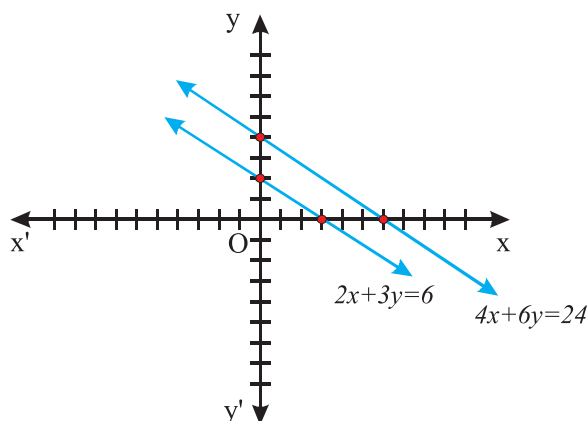
e.g. For $2x + 3y = 6$

x	0	3	-3
y	2	0	4

For $4x + 6y = 24$

x	0	6	-3
y	4	0	6

Draw the graph of the two equations on a graph paper as shown in the figure given below.



The learner may notice that the lines representing the equations are parallel i.e. the lines have no points in common. Therefore the two equations have no solution

Demonstration and Use: Complete the following table with the help of graphs drawn and the given pairs of equations:

S. No.	Pair of lines representing the equations are	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$
First pair of equations	Intersecting	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
Second pair of equations	Coincident	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Third pair of equations	Parallel	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$



Notes



Notes

Conclusion: Obtain the condition for two lines to be intersecting, coincident or parallel from

the above table by comparing the values of $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$. you will conclude that for

intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and

For parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Remark:

1. The learner may note that when a system of linear equations in two variables has solution (unique or infinite), the system is said to be consistent. When the system of equations has no solution it is said to be inconsistent.
2. The learner may verify the above conditions by taking some more examples

ACTIVITY 9



Notes

Title: To verify the relation between roots and coefficient of a quadratic equation.

Expected background knowledge:

- (i) Quadratic equation of the type $ax^2 + bx + c = 0, a \neq 0$
- (ii) Roots of a quadratic equation.

Objectives: After performing this activity, the learner will be able to establish the relation between the roots and coefficients of a quadratic equation.

Materials required: (i) Chart paper
(ii) Pencil and eraser

Preparation for the activity:

Write different quadratic equations with their roots.

e.g	roots
(i) $x^2 - 5x + 6 = 0$	2, 3
(ii) $x^2 - x - 6 = 0$	3, -2
(iii) $4x^2 - 8x + 3 = 0$	$3/2, 1/2$
(iv) $x^2 - 4x + 1 = 0$	$2 + \sqrt{3}, 2 - \sqrt{3}$
(v) $x^2 + 8x + 15 = 0$	-3, -5

Verify the roots by substituting in the corresponding quadratic equation

Demonstration and use

Prepare the following table on Chart paper

Sr. No.	Quadratic equation $ax^2 + bx + c = 0$	Roots α, β	Sum of roots $(\alpha + \beta)$	Product of roots $\alpha\beta$	-b/a	c/a
1.	$x^2 - 5x + 6 = 0$	$\alpha = 2, \beta = 3$	5	6	5	6
2.	$x^2 - x - 6 = 0$	$\alpha = 3, \beta = -2$	1	-6	1	-6
3.	$4x^2 - 8x + 3 = 0$	$\alpha = \frac{3}{2}, \beta = \frac{1}{2}$	2	$3/4$	2	$3/4$
4.	$x^2 - 4x + 1 = 0$	$\alpha = 2 + \sqrt{3}, \beta = 2 - \sqrt{3}$	4	1	4	1
5.	$x^2 + 8x + 15 = 0$	$\alpha = -3, \beta = -5$	-8	15	-8	15



Notes

Conclusion :

In a quadratic equation $ax^2 + bx + c = 0, a \neq 0$

$$\text{Sum of roots } (\alpha + \beta) = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of roots } \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Remark:

Result of this activity can be used in the following.

- (i) Forming quadratic equation when the roots are given.
- (ii) Finding the sum and product of roots of a quadratic equation, without actually finding the roots.

ACTIVITY 10



Title: To verify graphically that a quadratic polynomial can have at the most two zeroes.

Notes

Expected background knowledge:

1. Plotting the coordinates of a point on the graph.
2. Quadratic polynomial and its value at a given point.

Objectives: After performing this activity the learner will be able to:

- (i) find, how many zeroes a given quadratic polynomial has.
- (ii) find the zeroes of a quadratic polynomial.

Materials required:

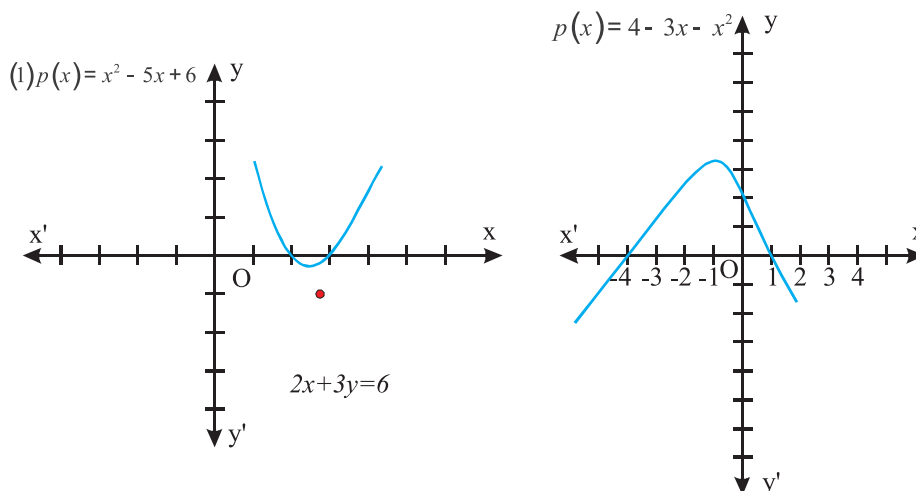
- (i) Graph papers (minimum 3)
- (ii) Geometry box
- (iii) Pencil and eraser.

Preparation for the activity:

Take quadratic polynomials of the form $ax^2 + bx + c$, $a \neq 0$ with different a, b, c. For example

- (i) $p(x) = x^2 - 5x + 6$
- (ii) $q(x) = -x^2 - 3x + 4$
- (iii) $r(x) = x^2 - 6x + 9$
- (iv) $g(x) = x^2 - 4x + 8$

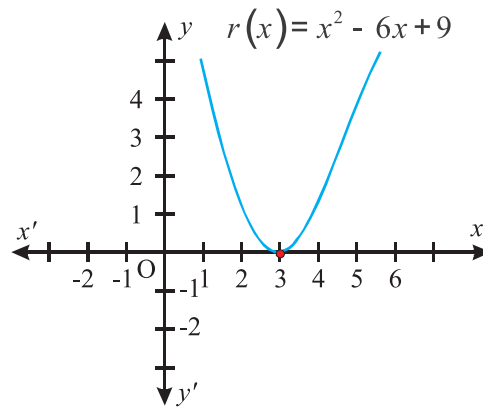
By finding the values of polynomials for different values of x, and plotting the coordinates $[x, p(x)]$ etc, draw the graphs of given polynomials.



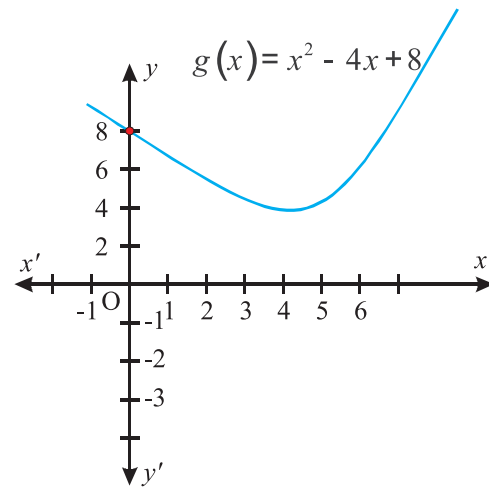


Notes

(iii)



(iv)



Demonstration and Use.

Polynomial	Graph Opening Upward / Downward	Number of Zeroes
$p(x) = x^2 - 5x + 6$	Upwards	Two
$q(x) = 4 - 3x - x^2$	Downwards	Two
$r(x) = x^2 - 6x + 9$	Upwards	One
$g(x) = x^2 - 4x + 8$	Upwards	Nil

Conclusion :

- (i) The graph of polynomial $ax^2 + bx + c$,
 - (a) opens upwards if $a > 0$
 - (b) opens downwards if $a < 0$
- (ii) Number of zeroes of a quadratic polynomial can be at most two.

ACTIVITY 11

Title: To verify that a given sequence is an A.P.

Expected Background knowledge: Knowledge of a sequence, definition of an A.P.

Objectives: After performing this activity, learner will be able to identify an arithmetic progression out of the given sequences.

Materials required:

- (i) Square papers with squares of size 1 cm x 1cm
- (ii) Pair of scissors
- (iii) Gum/fevicol
- (iv) Ruler, Pencil
- (v) Geometrical instruments.

Preparation for the activity: Consider the following sequences of positive numbers.

1, 4, 7, 10, 13, 16, ---
and 2, 3, 6, 10, 12, 15, 17, ---

For first sequence cut rectangular strips from coloured papers of different colours of width 1 cm and lengths 1 cm, 4cm, 7cm, 10 cm, --- Paste the coloured stripes in order on a squared paper as shown in the figure given below. [Fig.(i)]

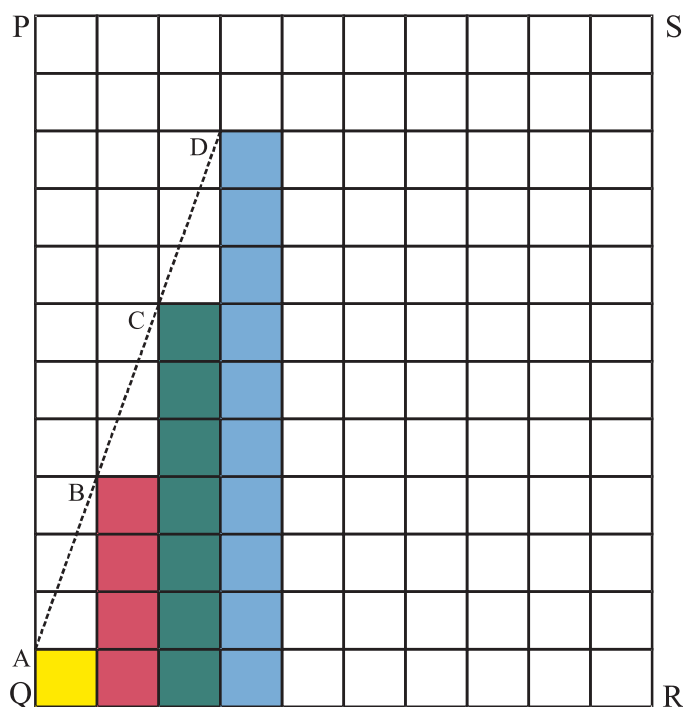


Fig. (i)

Notes





Notes

For the second sequence cut rectangular strips from coloured papers of different colours of width 1 cm and length 2 cm, 3 cm, 6 cm, 10 cm... Paste the coloured strips in order on a squared paper as shown in the figure given below. Fig. (ii)

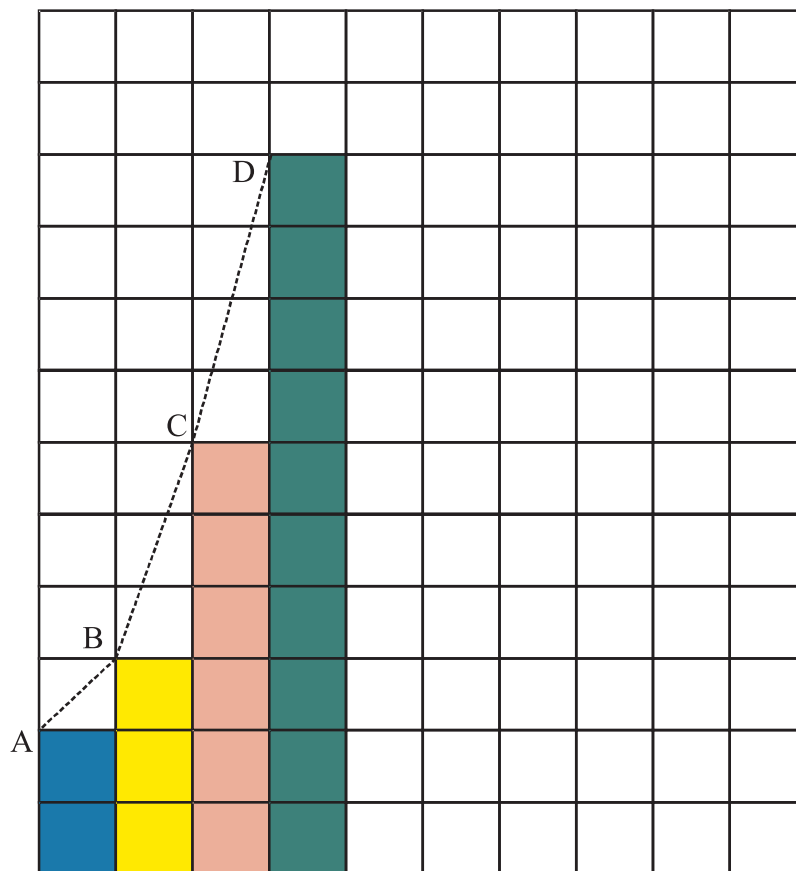


Fig. (ii)

Demonstration and Use

For the first sequence, the coloured strips form a ladder in which the difference between the heights of the adjoining strips is constant (Here it is 3 cm). For the second sequence the coloured strips form a ladder in which the difference between the heights of the adjoining strips is not constant.

Conclusion: For the first sequence which is an A.P. the difference between the heights of the adjoining strips of the ladder formed is constant and for the second sequence which is not an A.P. the difference between the heights of the adjoining strips of the ladder formed is not constant.

Hence, if the difference between the consecutive terms of a sequence is constant, then the given sequence is an A.P. otherwise it is not an A.P.

Note: In case of an A.P. the right hand top corners of the strips, when joined, form a straight line, which will not be the case, if the sequence is not an A.P.

ACTIVITY 12

Title: To find the Sum of first n odd natural numbers.

Expected Background knowledge: (i) Odd natural numbers

(ii) n th odd natural number can be written as $2n-1$.

Objectives: After performing this activity the learner will be able to generalize that the sum of first n odd natural numbers is given by n^2

i.e. $1+3+5+\dots+2n-1 = n^2$

Materials required:

- (i) White chart paper
- (ii) Scale, pencil and eraser
- (iii) Coloured ball point pens
- (iv) Pair of scissors
- (v) Geometrical instruments

Preparation for the activity:

- (i) Take a white chart paper and cut out a square of size 10cm x 10 cm from it and mark the boundary of the square.

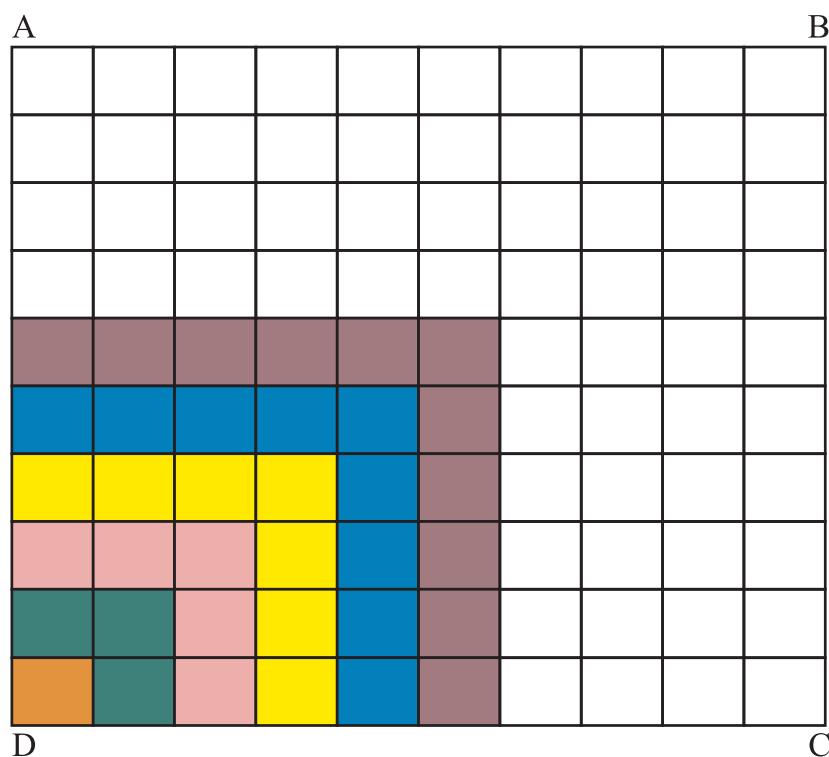


Fig. (i)

Notes





Notes

- (ii) Draw horizontal and vertical lines in the square to make small squares of size 1 cm x 1 cm as shown in the figure given below. (Fig.(i))
- (iii) Colour the small squares with different colours with the help of coloured pens as shown in the figure.

Demonstration and Use

Number of brown coloured small squares is one.

Number of green coloured small squares is three.

Number of red coloured small squares is five.

Number of yellow coloured small squares is seven.

Number of sky blue coloured small squares is nine.

Number of purple coloured small squares is eleven.

Now total number of small squares (brown = 1) in 1 cm x 1cm square is

$$1 = 1^2$$

Total number of small squares (brown + green = (1+3) in 2cm x 2cm square is $4 = 2^2$

Total number of small squares (brown + green + red = (1+3+5) in 3cm x 3cm square is $9 = 3^2$

Total number of small squares (brown + green + red + yellow = (1+3+5+7) in 4cm x 4cm square is $16 = 4^2$

Total number of small squares (brown + green + red + yellow + sky blue + purple = (1+3+5+7+9) in 5cm x 5cm square is $25 = 5^2$

Total number of small squares (brown + green + red + yellow + sky blue + purple = 1+3+5+7+9+11) in 6cm x 6cm square is $36 = 6^2$

and so on

Conclusion:

Proceeding in thus way we observe that the total number of small squares in an n cm x ncm square is $1+3+5+7+9+11+ \dots + (2n-1) = n^2$

Hence we can say that the sum of first n odd natural numbers is n^2 .

ACTIVITY 13

Title: To find the sum of first n natural numbers.

Expected Background knowledge: (i) Natural numbers and operations on them.
(ii) Area of a square and a rectangle.

Objectives: After performing the activity, learner will be able to find the sum of first, n natural numbers.

Materials required:

- (i) Chart paper
- (ii) Scale, Pencil and Eraser
- (iii) Colour box/ Coloured ball point pens
- (iv) Geometrical Instruments.
- (v) Pair of scissors/cutter

Preparation for the activity:

- (i) Cut out a chart paper ABCD of size $10\text{cm} \times 11\text{cm}$ and mark its boundary.
- (ii) Draw horizontal and vertical lines in the above rectangular paper to mark squares of size $1\text{cm} \times 1\text{cm}$ as shown in the figure below. [Fig. (i)]

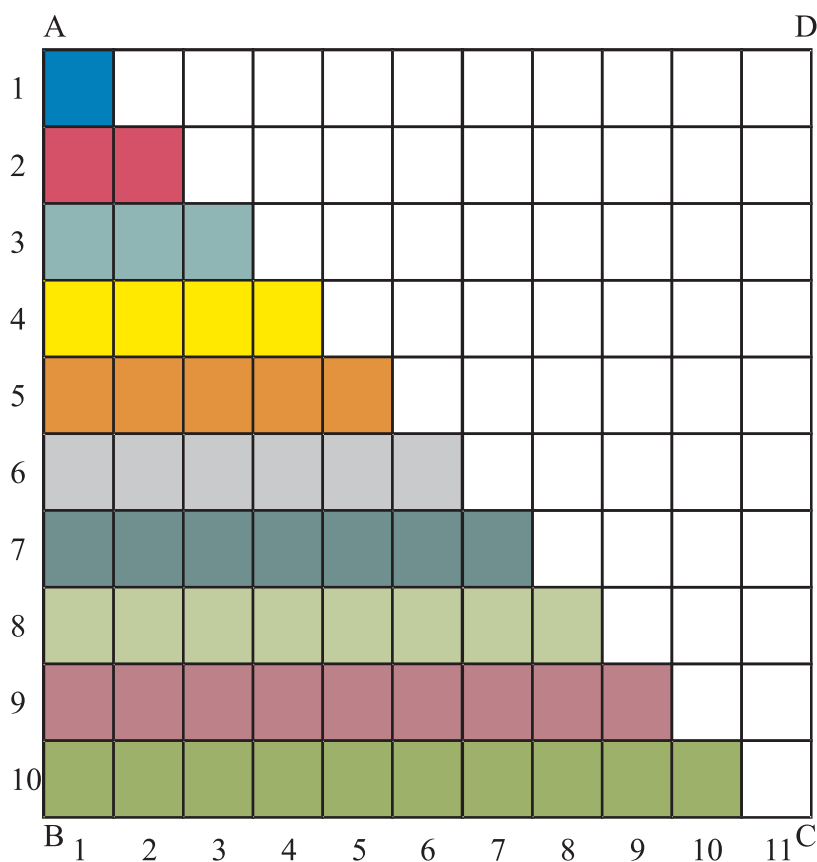


Fig. (i)



Notes



Notes

- (iii) Mark the squares as, 1,2,3, 10, along vertical line and 1,2,3,.....11, along horizontal line.
- (iv) Starting from the left top corner colour the square of size 1cm x 1cm, rectangle of size 2cm x 1cm, rectangle of size 3cm x 1cm,.....with different colours as shown in the figure.

Demonstration and Use

- (i) Area of coloured portion in the figure

$$= \text{Area of square of size } 1\text{cm} \times 1\text{cm} + \text{area of rectangle of size } 2\text{cm} \times 1\text{cm} + \dots\dots\dots + \text{area of rectangle of size } 10\text{cm} \times 1\text{cm}$$

$$= (1 + 2 + 3 + \dots\dots\dots + 10) \text{ cm}^2$$
- (ii) Area of coloured region = $\frac{1}{2}$ area of rectangle ABCD.
- (iii) Area of rectangle ABCD = $10 \text{ cm} \times 11 \text{ cm}$

$$= (10 \times 11) \text{ cm}^2$$
- (iv) Area of coloured region = $(\frac{1}{2} \times 10 \times 11) \text{ cm}^2$
 From (i) and (iv) we have

$$1+2+3+\dots\dots\dots 10 = (\frac{1}{2} \times 10 \times 11).$$

Proceeding in this manner and generalizing the result we get

$$1 + 2 + 3 + \dots\dots\dots + n = \frac{1}{2} [n(n+1)]$$

Conclusion:

\therefore Sum of first n natural numbers is

$$= \frac{n(n+1)}{2}$$

ACTIVITY 14



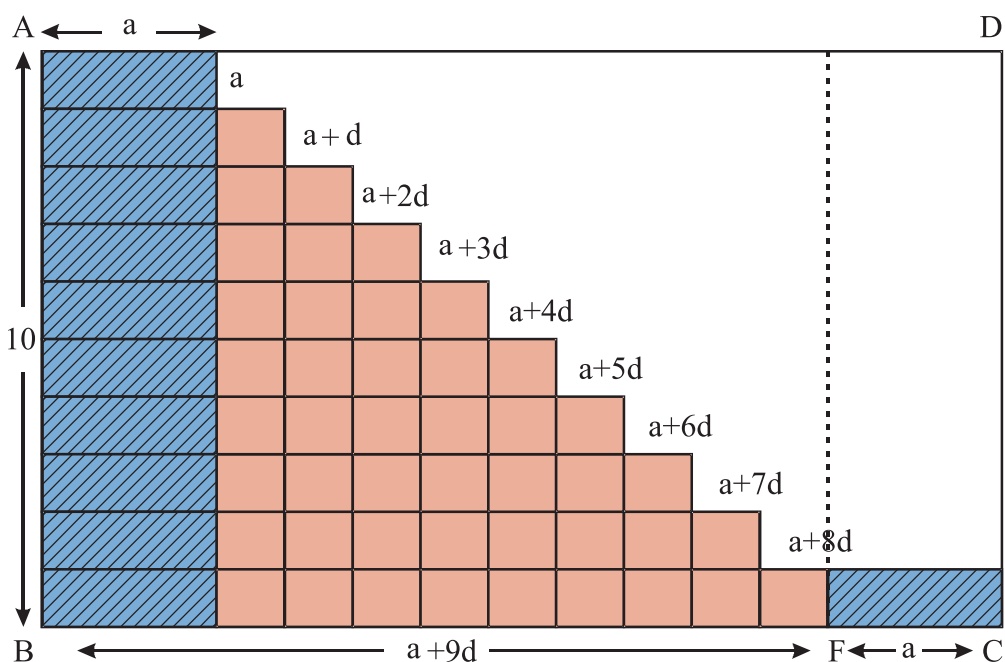
Title: To find the sum of first n terms of an arithmetic progression (A.P.).

Expected Background knowledge: Knowledge of arithmetic progression.

Objectives: After performing this activity the learner will be able to find the sum of any number of terms of an A.P.

Materials required:

- (i) Plastic strips
- (ii) Coloured chart paper
- (iii) Thermocol sheets
- (iv) Fevicol
- (v) Pair of scissors
- (vi) Scale, pencil and eraser



Preparation for the activity :-

- (i) Take a rectangular thermocol sheet ABCD.
- (ii) Cut some plastic strips of equal fixed length, denoted by a and some others of equal length denoted by d .
- (iii) Arrange and paste both types of strips so as to get terms, $a, a+d, a+2d, \dots, a+9d$ placed at unit distance apart and arrange along the rectangle, as shown above
- (iv) The last strip ends in F on BC , extend F To C by a fixed length a .



Notes

Demonstration and Use.

- (i) The first strip is of length a
- (ii) Second strip is of length $a+d$
- (iii) Third strip is of length $a+2d$
- (iv) Tenth strip is of length $a + 9d$.
- (v) Strips arranged look like a stair case
- (vi) The sum of above arithmetic progression

$$= a + (a+d) + (a+2d) + \dots + (a+9d)$$

$$= 10a + 45d$$

$$= 5 (2a+9d) = \frac{1}{2} \cdot 10 \cdot (2a + 9d) = \frac{1}{2} \cdot 10 [2a + (10-1)d]$$

$$= \frac{1}{2} (\text{Area of rectangle ABCD, where length BC} = 2a + 9 \text{ and breadth is 10 units})$$

Conclusion

If the arithmetic progression is $a, a+d, a+2d, \dots, a+(n-1)d$, then the sum of its first n terms

$$= \frac{n}{2} [2a + (n-1)d]$$

ACTIVITY 15



Notes

Title: To verify that the sum of the angles of a triangle is 180°

Expected background knowledge: (i) Angles and Triangles
(ii) Construction of angles and triangles

Objectives: After performing this activity, the learner will be able to

- (i) verify and demonstrate that the sum of the angles of a triangle is 180° .
- (ii) find the measure of an angle of a triangle when the measure of other two angles are given.

Materials required:

- (i) Coloured glazed papers
- (ii) Coloured Paper (card) board
- (iii) Scale
- (iv) Pencil
- (v) Eraser
- (vi) Fevicol
- (vii) Pair of scissors/cutter

Preparation for the activity:-

- (i) Take the orange coloured Cardboard
- (ii) Draw a triangle ABC on a thick white paper. Cut out the triangular region and paste it on the cardboard. [Fig.(i)]

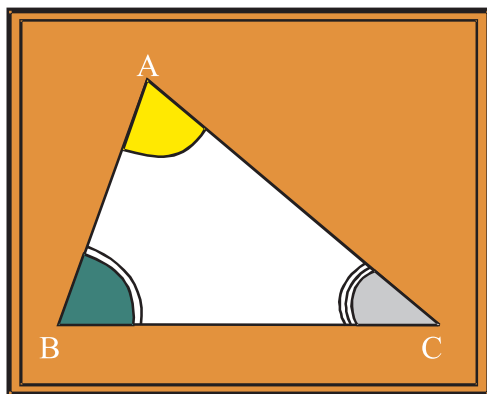


Fig. (i)

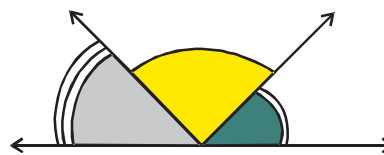


Fig. (ii)



Notes

Demonstration and Use

- (i) Cut off angles BAC, ACB and CBA from the triangular portion and colour them yellow, grey and green respectively
- (ii) Paste these cut-out angles on a sheet of paper, as shown in Fig. (ii)
- (iii) It can be seen that the three angles together form a straight angle, showing thereby that the sum of the angles of a triangle is 180°

Conclusion :

The sum of the angles of any triangle is 180°

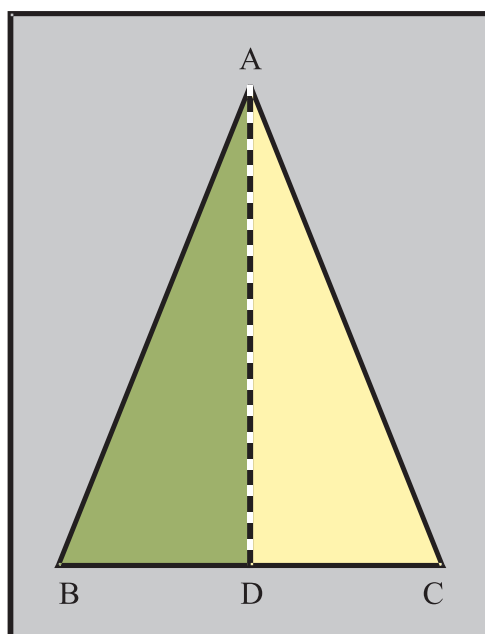
ACTIVITY 16**Notes**

Title: To verify that the angles opposite to equal sides of a triangle are equal.

Expected background knowledge:

- (i) Construction of triangles
- (ii) Congruency of triangles
- (iii) Paper folding and superposition

Objectives: After completion of the activity, the learner will be able to demonstrate this concept and use it in proving the problems requiring this knowledge



Materials required:

- (i) Grey Card board sheet of size 25 cm x 30cm.
- (ii) Pencil
- (iii) Eraser
- (iv) Fevicol
- (v) Pair of compasses
- (vi) Scale
- (vii) Pair of scissors/cutter



Notes

Preparation for the activity:

- (i) On a bigger white board, paste a grey cardboard of size 25cm x 30cm
- (ii) Draw a $\triangle ABC$, in which $AB=AC$, on the cardboard and make a copy of $\triangle ABC$ on a white sheet
- (iii) Of the $\triangle ABC$, made on white sheet, find the median AD using paper folding
- (iv) Colour the two halves in different colours and paste the triangle on the triangle drawn on the board, as shown in the figure
- (v) Keep the fold AD loose on the board

Demonstration and Use

- (i) Fold the pasted triangle ABC along the median AD
- (ii) See that point C falls on B and AC falls along AB . You can see that CD falls along BD , showing that $\angle ABC = \angle ACB$

Conclusion:

Angles opposite to equal sides of a triangle are equal

ACTIVITY 17

Title: To verify the Mid-Point Theorem

Expected background knowledge:

- (i) Knowledge of parallel lines
- (ii) Knowledge of parallelograms
- (iii) Criteria for a quadrilateral to be a parallelogram

Objectives: After performing this activity, the learner will be able to:

- (i) recognize the importance of the result
- (ii) use it wherever needed for proving other results

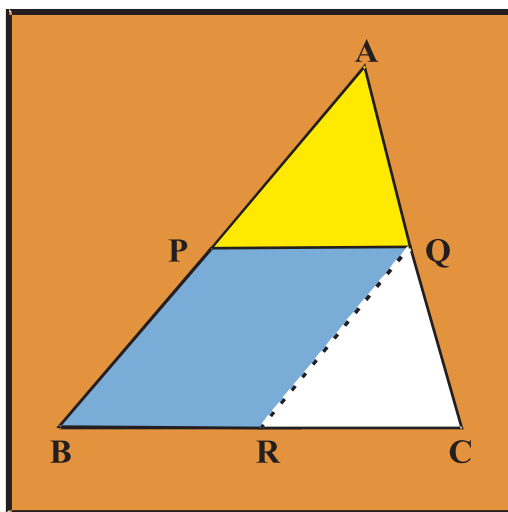


Fig. (i)

Materials required:

- (i) Orange coloured thick board
- (ii) Coloured and white sheets (papers)
- (iii) Gum/Fevicol
- (iv) Pair of scissors/ Cutter
- (v) Pair of compasses
- (vi) Pencils and sketch pens
- (vii) Scale and Eraser

Notes





Notes

Preparation for the activity:

- (i) Cut out a squared from the orange thick board of size 20cm x 20 cm
- (ii) From a sheet of paper, cut a $\triangle ABC$
- (iii) Find the mid-points P and Q of the sides AB, and AC by paper folding and join P and Q by making the crease PQ
- (iv) Cut the $\triangle APQ$ from $\triangle ABC$ and superpose AQ over QC so that QP falls along CB as shown in the figure

Demonstration and Use

$$\triangle APQ \cong \triangle QRC$$

$$AP = QR = \frac{1}{2} AB = PB$$

and $\angle APQ = \angle QRC = \angle PBC; \angle PBC + \angle QRB = 180^\circ$

PQRB is a parallelogram

$$PQ = BR = \frac{1}{2} BC$$

and $PQ \parallel BC$

Conclusion:

The line segment joining the mid points of any two sides of a triangle is parallel to the third side and is half of it.

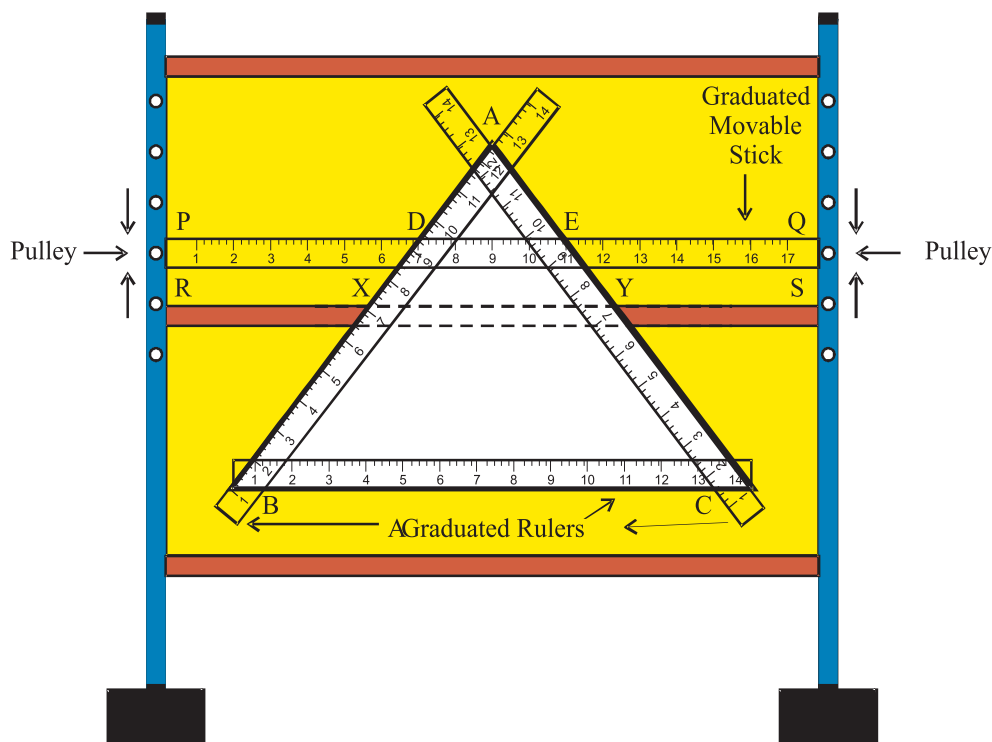
ACTIVITY 18

Title: To verify Basic Proportionality Theorem

Expected background knowledge:

- (i) Knowledge of parallel lines and their construction
- (ii) Knowledge of triangles, triangular regions and their construction
- (iii) Concept of ratio and proportion.

Objectives: After completion of this activity the learner will be able to demonstrate the theorem and apply it in situations, where the conditions of the theorem are fulfilled.



Materials required:

- (i) Stands with grooves so that they can keep any rod, fixed on these through pulleys, straight
- (ii) Wooden board with yellow paint on it
- (iii) A triangular region (of thick paper)
- (iv) Graduated scales (At least 4)
- (v) Screws and screw driver
- (vi) Glue/Fevicol



Notes



Notes

- (vii) Sketch Pens
- (viii) Pulleys
- (ix) Pair of Scissors.
- (x) Nails of different sizes (to fix pulleys in grooves)

Preparation for the activity:

- (i) Take the yellow wooden board and fix it with the help of screws on the stands
- (ii) Fix pulleys, on the stands with the help of nails.
- (iii) Cut a triangle from the thick paper and name it $\triangle ABC$ and paste it on the wooden board
- (iv) Fix three graduated rulers along the sides of $\triangle ABC$, with base BC
- (v) Fix the straight ruler, parallel to the base of the triangle, using pulleys and nails in such a way that the ruler can be moved up and down parallel to base of the $\triangle ABC$

Demonstration and Use

A.

- (i) Fix the position of the horizontal graduated ruler PQ and read the distances AD, BD and AE, CE along the rulers along the sides AB and AC respectively
- (ii) Also read the length of DE and BC
- (iii) Calculate $\frac{AD}{BD}, \frac{AE}{CE}$
- (iv) You will find that $\frac{AD}{BD} = \frac{AE}{CE}$

B. Fix the position of the horizontal graduated ruler at R and S and again find the ratios

$$\frac{AX}{XB} \text{ and } \frac{AY}{YC},$$

You will again find that all the ratios are equal, proving Basic Proportionality Theorem

Observation:

You can also check that

$$(i) \quad \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

and $(ii) \quad \frac{AX}{AB} = \frac{AY}{AC} = \frac{XY}{BC}$

Conclusion: If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it will divide the other two sides in the same ratio.

ACTIVITY 19

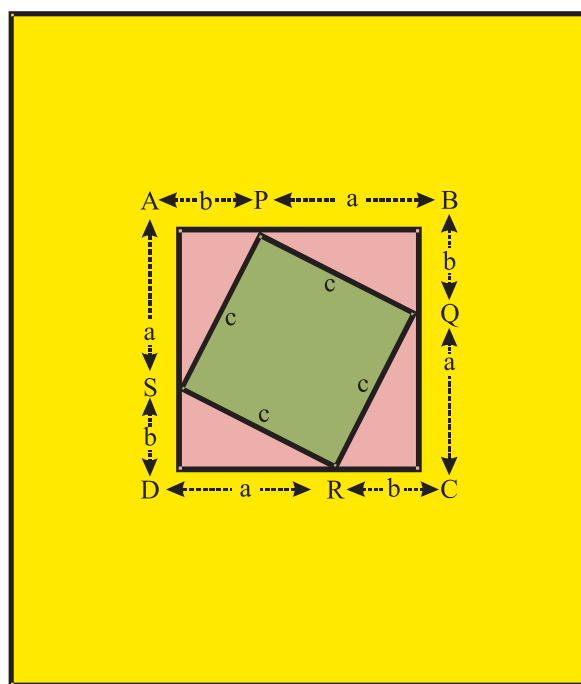
Title: To verify Pythagoras Theorem

Expected Background knowledge:

- (i) Knowledge about triangles and their types
- (ii) Similarity in triangles
- (iii) Idea of ratio and proportion

Objective: After performing this activity, the learner will be able to

- (i) identify right triangles from a given number of triangles
- (ii) use its result, wherever required, to simplify/or solve problems



Materials required :

- (i) Yellow Card Board
- (ii) Various Coloured Papers
- (iii) Pens/markers
- (iv) Fevicol
- (v) Pencil/Sharpner

Notes





Notes

- (vi) Eraser
- (vii) Drawing Pins

Preparation for the activity:

- (i) Cut off a yellow Card Board of size $(10\text{cm} \times 10\text{cm})$
- (ii) On an orange sheet of paper draw a square of side $a + b$, (where $a = 3\text{cm}$ and $b = 1\text{cm}$) and name it ABCD
- (iii) On the sides AB, BC, CD and DA, take points P, Q, R and S such that $AP = BQ = CR = DS = b$ (1cm)
- (iv) Paste the square ABCD on the yellow board
- (v) On the orange square, paste a square PQRS of side PQ (or $QR = c$, say) made on a green sheet

Demonstration:

- (i) Area of square ABCD $= (a + b)^2$ sq. units
 $= (a^2 + b^2 + 2ab)$ sq. units
- (ii) Area of square PQRS $= c^2$
- (iii) The four orange triangular regions are equal in area as they are congruent. (by SSS)
 Their joint area $= 4 \left[\frac{1}{2} ab \right] = 2ab$

Now, Area of square ABCD = Area of sq. PQRS + Area of 4 orange triangles

$$a^2 + b^2 + 2ab = c^2 + 2ab$$

$$\Rightarrow c^2 = a^2 + b^2$$

Which verifies Pythagoras theorem

Conclusion:

- (i) In any right triangle, the square on the hypotenuse is equal to sum of the squares on the other two sides
- (ii) The lengths of the three sides of a right triangle are called “Pythagorean Triplets”. For example
 a) 3,4,5 b) 5,12,13 c) 7, 24,25 etc.
- (iii) The Converse of Pythagoras theorem also holds i.e. for a triangle if $c^2 = a^2 + b^2$, then the triangle is right angled at C, where $BC = a$, $AC = b$ and $AB = c$.

ACTIVITY 20



Notes

Title : To verify that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Expected background knowledge:- (i) Areas of triangles (ii) concept of similarity

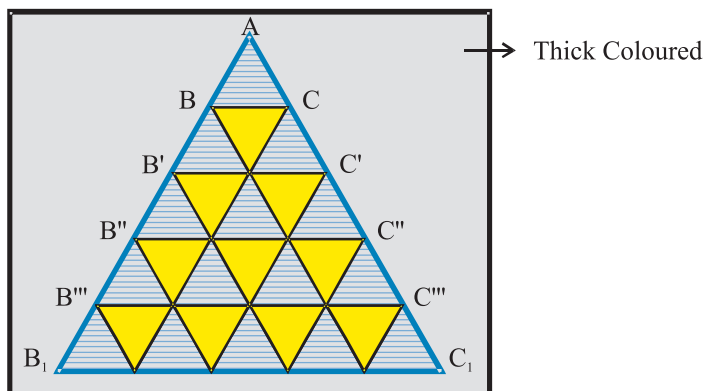
Objectives:- After performing the activity, the learner will be able to apply the result wherever required, in problems of geometry.

Materials required :

1. Thick coloured sheet
2. Coloured and lined Papers
3. Pair of scissors
4. Fevicol
5. Sketch Pens
6. Scale
7. Compass
8. Drawing Pins
9. Markers

Preparation for the activity :-

- (i) Take the thick coloured sheet and on it draw an equilateral triangle of side 5cm (AB_1C_1) with the help of scale, compass and marker
- (ii) Divide each of the sides in 5 equal parts and through these divisions draw lines parallel to the sides of the triangle, creating the figure containing 25 equilateral (and congruent) triangles
- (iii) Paste yellow papers and lined papers on the triangles as shown in the figure





Notes

Demonstration and Use: From triangle ABC and triangle $AB'C'$ we have

$$\frac{ar(ABC)}{ar(AB'C')} = \frac{1}{4} = \left(\frac{1}{2}\right)^2 = \frac{BC^2}{B'C'^2}$$

Again,

$$\frac{ar(ABC)}{ar(AB''C'')} = \frac{1}{9} = \left(\frac{1}{3}\right)^2 = \left(\frac{BC}{B''C''}\right)^2 = \frac{BC^2}{B''C''^2}$$

Similarly,

$$\frac{ar(ABC)}{ar(AB'''C''')} = \frac{1}{16} = \frac{(BC)^2}{B'''C'''^2}$$

and

$$\frac{ar(ABC)}{ar(AB_1C_1)} = \frac{1}{25} = \left(\frac{1}{5}\right)^2 \frac{BC^2}{B_1C_1^2}$$

Conclusion : Ratio of the areas of two similar triangles = Ratio of the squares of their corresponding sides

ACTIVITY 21

Title : To find the area of a circle

Expected background knowledge:-

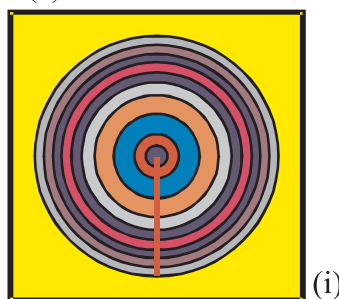
- (i) Concept of area
- (ii) Circle and its related terms

Objectives:- After the start and finish of activity, the learner will be able to quote the area of a circle correctly and use it wherever needed.

Materials required:

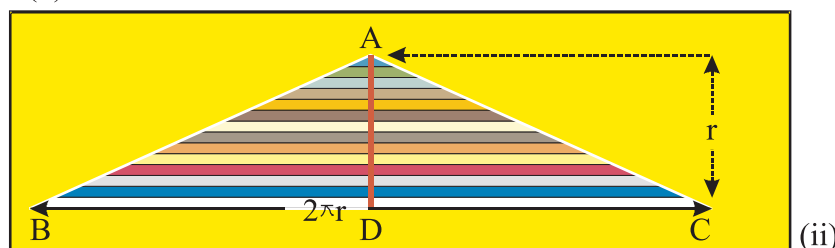
- (i) Threads of different colours
- (ii) Pair of compasses
- (iii) Pencil
- (iv) Pair of scissors.
- (v) Fevicol
- (vi) Yellow Thick cardboard

2(a)



(i)

2(b)



(ii)

Preparation for the activity: -

- (i) Cut-off the yellow thick board of size 15cm x 15cm
- (ii) Using pair of compasses, draw concentric circles as shown in fig. (i)
- (iii) Arrange coloured Threads on the concentric circles as shown in (i)
- (iv) Cut-off the circles from the innermost thread to the outermost thread and arrange them as shown in fig. (ii) as a triangle



Notes



Notes

Demonstration and Use

Let r be the radius of outermost circle

\therefore Base BC of triangle ABC has length = $2\pi r$ units

Length of AD, the altitude of triangle ABC = r units

Area of circle = Area of triangle ABC = $\frac{1}{2} BC \times AD$

$$= \frac{1}{2} (2\pi r)(r) \text{ sq. units}$$

$$= \pi r^2 \text{ sq. units}$$

Observation:

- (i) The cut-outs laid down in (ii) form an approximate triangle
- (ii) With no wastage, area of circle = Area of cut out threads laid down

Conclusion: Area of a circle of radius r is πr^2 .

ACTIVITY 22



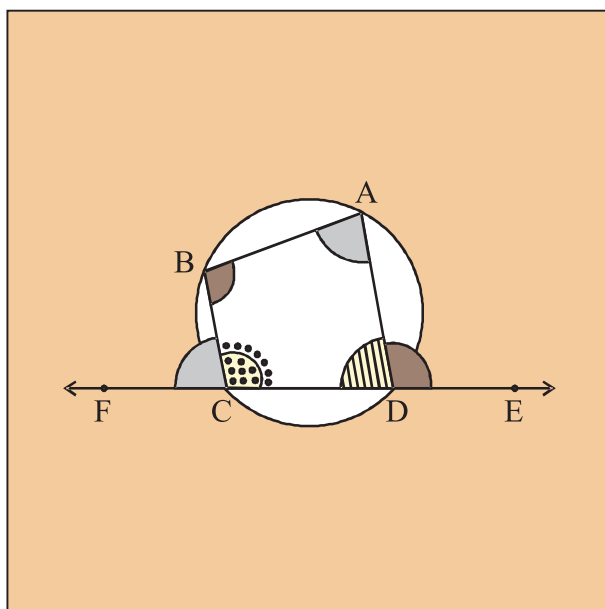
Notes

Title : To demonstrate that the opposite angles of a cyclic quadrilateral are supplementary

Expected Background knowledge : Concept of a cyclic quadrilateral

Objectives:- To prepare a model to demonstrate the above

Materials required: Plyboard, coloured Cardboard, drawing pins, glazed paper, sketch pens, Fevicol, pair of scissors.



Preparation for the activity: -

- (i) Cut out a circle of radius (5cm) on the Cardboard sheet and paste Yellow glazed paper on it
- (ii) Draw a cyclic quadrilateral ABCD on the yellow glazed paper and extend the side CD to both sides to E and F forming exterior angles ADE and BCF
- (iii) Cut-out a portion of $\angle A$ and $\angle B$ and paste on the exterior angles BCF and ADE respectively as shown in the figure
- (iv) You can see that $\angle D + \angle ADE = 180^\circ \Rightarrow \angle D + \angle B = 180^\circ$

And

$$\angle C + \angle BCF = 180^\circ \Rightarrow \angle A + \angle C = 180^\circ$$



Notes

Demonstration and Use

This model can be used to verify that:

- (i) Opposite angles of a cyclic quadrilateral are supplementary
- (ii) Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle

ACTIVITY 23



Notes

Title: To verify that equal chords of congruent circles subtend equal angles at the centres.

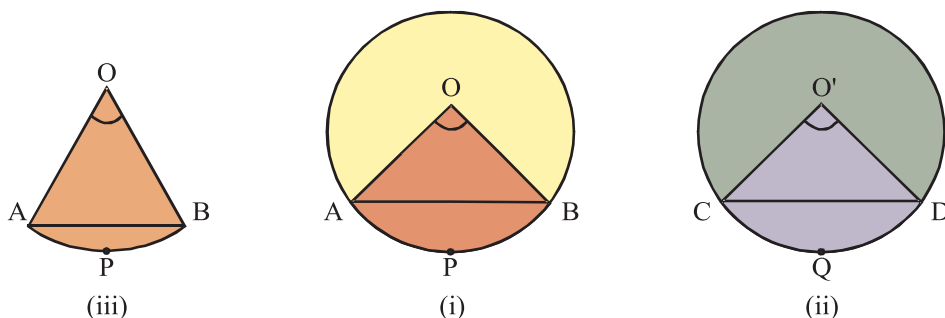
Expected Background knowledge:

- (i) Terms related to circles
- (ii) Congruency of triangles

Objective:- After performing this activity, the learner should be able to state and verify the result of the activity.

Materials required :

- (i) Coloured papers
- (ii) Sketch pens
- (iii) Pencil and scale
- (iv) Eraser
- (v) Pair of scissors
- (vi) Fevicol



Preparation for the activity: -

- (i) Draw two congruent circles (of same radii), one on yellow paper and the other on green paper with centres O and O'
- (ii) On yellow paper, draw a chord AB and on green paper, draw a chord CD of length equal to the length of AB.
- (iii) Join AO and BO and CO' and DO'



Notes

Demonstration and Use

- (i) Cut out sector AOBP from the circle on yellow paper or make a replica of it and superpose it on the sector with circle on the green sheet such that AOBP falls on $CO'DQ$.
- (ii) You will find the sector AOBP completely covers sector $CO'DQ$ which shows that $\angle AOB = \angle CO'D$

Conclusion: This proves that equal chords of congruent circles subtend equal angles at the centres

Application :

You can also verify that the lengths of arcs of circles which subtend equal angles at the centre are also equal.

ACTIVITY 24



Notes

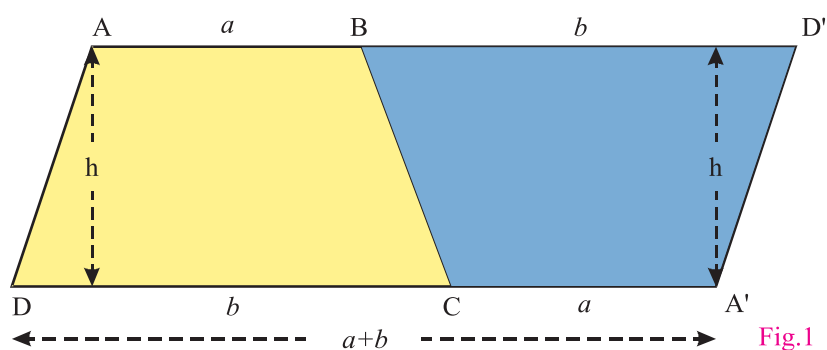
Title : To find the area of a trapezium

Expected Background knowledge: Recognition of a trapezium and knowledge of terms related to it.

Objective:- After performing this activity the learner should be able state the formula and find the areas of different trapezia.

Materials required :

- (i) Coloured papers
- (ii) Geometry box
- (iii) Fevicol
- (iv) Pair of scissor
- (v) Thermocol
- (vi) Hardboard



Preparation for the activity: -

- (i) Take a piece of hardboard
- (ii) Cut two congruent trapezia of parallel sides a and b from yellow and blue papers
- (iii) Paste the two trapezia on the hardboard as shown in Fig. 1.



Notes

Demonstration and Use :

It can be easily seen that the two trapezia together form a parallelogram of base $(a+b)$ and height h

Area of parallelogram $AD'A'D = h(a+b)$

$$\therefore \text{Area of trapezium } ABCD = \frac{1}{2}[(a+b) \times h]$$

Conclusion: The area of a trapezium is equal to $\frac{1}{2}$ [(sum of its parallel sides) \times (perpendicular distance-between them)]

ACTIVITY 25



Notes

Title: To find the total surface area of a cube

Expected Background Knowledge:

- (i) Knowledge and recognition of solids
- (ii) Characteristics of a cube

Objectives:- After performing the activity the learner should be able to state the formula of surface area of a cube and calculate it when asked for

Materials required :

- (i) White Paper
- (ii) Pencil and eraser
- (iii) Geometrical instruments
- (iv) Sketch pens
- (v) Scale
- (vi) Fevicol

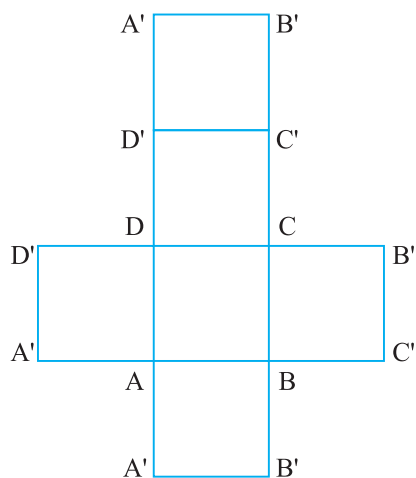


Fig. (i)

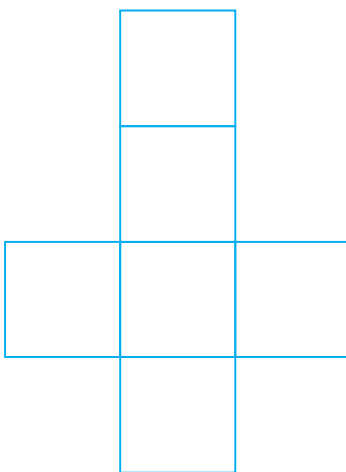


Fig. (ii)

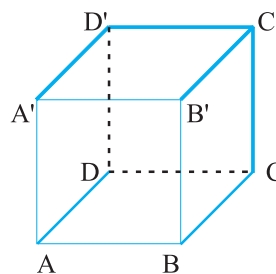


Fig. (iii)



Notes

Preparation for the activity:

- (i) Make two rectangles of dimensions 8 cm x 2cm and 6 cm x 2cm, intersecting along a common square ABCD
- (ii) Take square ABCD as base, and draw lines showing six different squares as shown in Fig.(i)
- (iii) Keep the square ABCD as base, and fold other squares along the edges, as shown in Fig. (ii)

Demonstration and Use :

Fold different squares along the folds to get a cube as shown in Fig. (iii) The total surface area of the cube is sum of the areas of 6 squares which is $6 (\text{side})^2$

Conclusion: The surface area of a cube is $6 (\text{side of the cube})^2$

ACTIVITY 26



Notes

Title : To find the formula for curved surface area of a cone-using the formula for the area of sector of a circle

Expected Background knowledge:

- (i) Idea of a cone
- (ii) Area of sector of a circle
- (iii) Length of arc of sector of a circle.

Objective:- After performing the activity, the learner should be able to state the formula for the curved surface area of a cone and calculate it when asked for.

Materials required :

- (i) Thick white sheet
- (ii) Red paper
- (iii) Sketch pens
- (iv) Pair of scissors/cutter
- (v) Fevicol

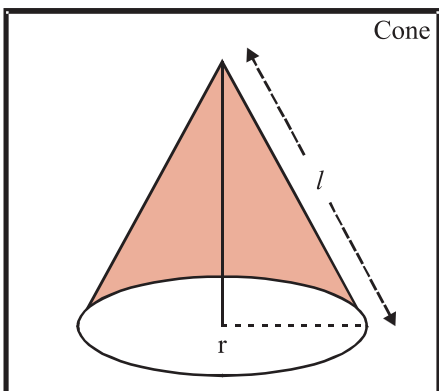


Fig. (i)

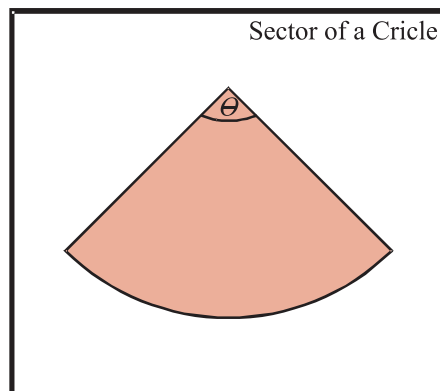


Fig. (ii)

Preparation for the activity: -

- (i) Take a cone made of thick red paper of slant height l and radius r
- (ii) Cut the curved surface of cone along any slant height by a cutter
- (iii) Paste the cut-out in the form of sector of a circle, of radius l on a white sheet of paper [See figure (ii) above]



Notes

Demonstration and Use :

Let θ be the central angle of circle of which sector is a part. It can be seen that circumference of base of cone forms the arc length of the sector

$$\therefore 2\pi r = 2\pi l \cdot \frac{\theta}{360} \Rightarrow \theta = 360^\circ \cdot \frac{r}{l}$$

Area of sector = Curved surface area of cone

$$\text{Area of sector} = \pi l^2 \cdot \frac{\theta}{360^\circ} = \left(\frac{\pi l^2}{360^\circ} \right) \left(360^\circ \frac{r}{l} \right) = \pi rl$$

Curved surface area of a cone = πrl

Conclusion: The curved surface area of a cone = π (radius of base) \times (slant height of the cone)

ACTIVITY 27



Notes

Title: To find the relationship among the volumes of a right circular cone, a right circular cylinder and a hemisphere of same radii and same heights.

Expected Background knowledge: Knowledge of solids – cone, cylinder and hemisphere

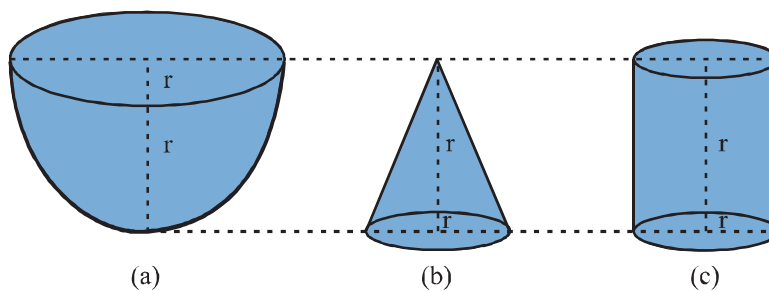
Objectives:- After performing the activity the learner is able to find the ratio of volumes of right circular cone, right circular cylinder and a hemisphere of same base radii and same heights.

Materials required :

- (i) Plastic sheet
- (ii) Plastic ball
- (iii) Fevicol
- (iv) Sketch pens
- (v) Sand

Preparation for the activity: -

- (i) Take a plastic ball of radius 10cm and cut it into two-halves, so as to get a hemisphere



- (ii) From the plastic sheet, make a right circular cone of base radius 10cm and height 10cm
- (iii) Similarly, from the plastic sheet make a right circular cylinder of base radius 10cm and height 10cm

Demonstration and Use :

- (i) Fill the cone with sand and pour it twice into the hemispherical shell. You will see the shell is full to the brim with sand
- (ii) Fill the cone with sand and pour it 3 times in the cylinder. You will again see that the cylinder is full to the brim



Notes

$$(iii) \quad \text{Volume of cone} = \frac{1}{3} \pi r^2 . h = \frac{1}{3} \pi r^2 . r (\because h = r) = \frac{1}{3} \pi r^3 .$$

$$\therefore \quad \text{Volume of hemisphere} = \frac{1}{3} \pi r^3 \times 2 = \frac{2}{3} \pi r^3 .$$

$$\text{Volume of cylinder} = \frac{1}{3} \pi r^3 \times 3 = \pi r^3$$

$$\therefore \quad \text{Required ratio} \quad \frac{1}{3} \pi r^3 : \pi r^3 : \frac{2}{3} \pi r^3 \Rightarrow 1 : 3 : 2$$

ACTIVITY 28



Notes

Title: To verify the identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Expected Background Knowledge:

- (i) Volume of a cube and cuboid.

Objectives:- After performing this activity the learner will be able to verify the identity and use it, wherever needed.

Materials required :

- (i) Acrylic sheet
- (ii) Wooden board
- (iii) Sketch pens
- (iv) Glazed paper
- (v) Pair of scissors
- (vi) Gum/Fevicol

Preparation for the activity: -

- (i) Make a cuboid of size $(a-b) \times a \times a$ cubic units [here say $a=3$ units, and $b=1$ unit] using wooden board as shown in Fig. 1(a)
- (ii) Make another cuboid of size $(a-b) \times a \times b [2 \times 3 \times 1]$ cubic units, using the same wooden board, as shown in Fig. 1(b)
- (iii) Make one more cuboid of size $(a-b) \times b \times b [2 \times 1 \times 1]$ cubic units using the same wooden board, as shown in Fig. 1(c)
- (iv) Make a cube of size $b \times b \times b [1 \times 1 \times 1]$ cubic units as shown in Fig. 1(d), from the same wooden board.
- (v) Make a cube of size $a \times a \times a [3 \times 3 \times 3]$ cubic units, from the acrylic sheet, as shown in Fig. 1(e)



Notes

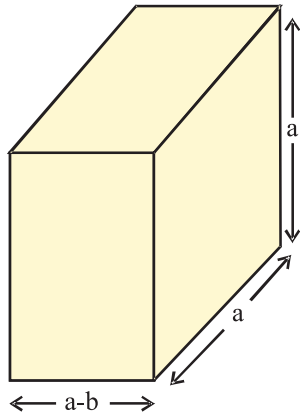


Fig.1(a)

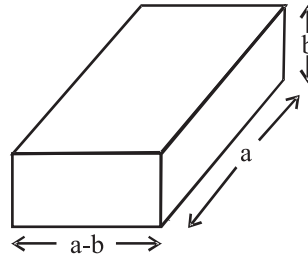


Fig.1(b)

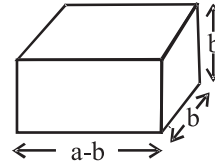


Fig.1(c)

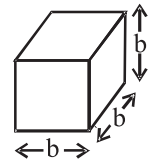


Fig.1(d)

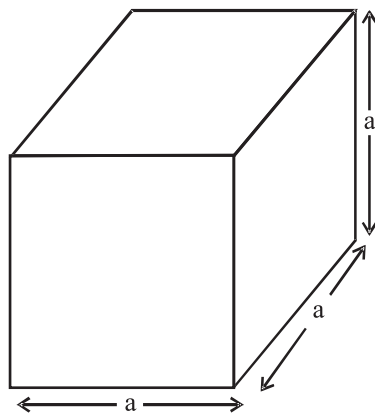


Fig.1(e)

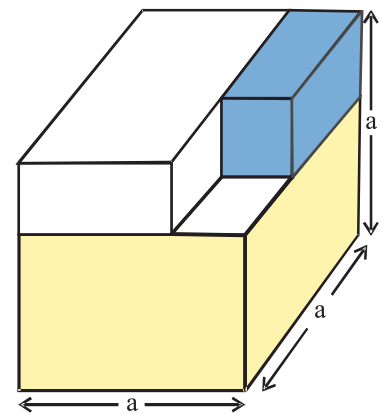


Fig.1(f)

Demonstration and Use :

Assemble all the three cuboids in such a way that they give a cube of size $3 \times 3 \times 3$ cubic units.

By suitable arrangements of cubes and cuboids, the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ can be verified as follows:

$$\begin{aligned} a^3 &= (a - b) \times a \times a + (a - b) \times a \times b + (a - b) \times b \times b + b \times b \times b \\ &= (a - b)(a^2 + ab + b^2) + b^3 \\ \Rightarrow a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \end{aligned}$$

So, the model activity can be used to verify the algebraic identity.

Conclusion: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

ACTIVITY 29



Notes

Title: To draw a triangle equal in area to a parallelogram

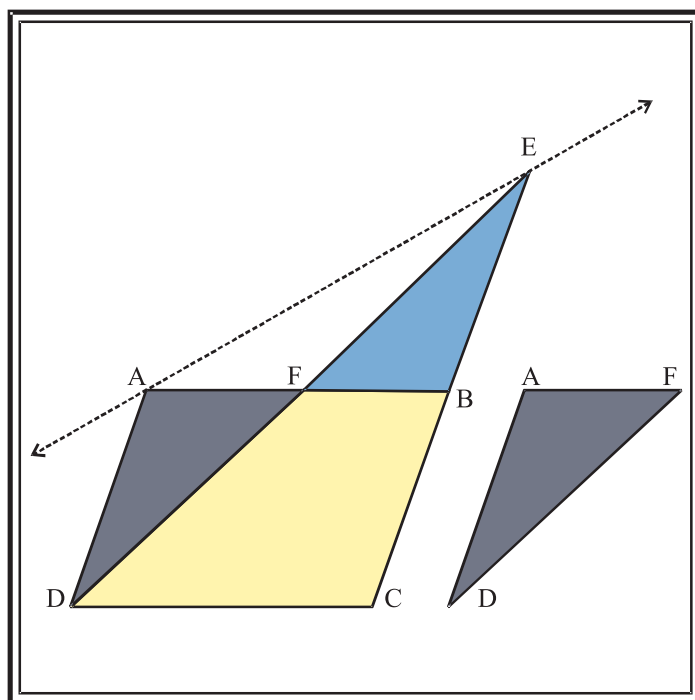
Expected Background Knowledge:

- (i) Concept of area
- (ii) Parallelograms and triangles
- (iii) Area in relation to parallelograms and triangles

Objectives:- After performing the activity, the learner will be able to draw different triangles equal in area to a parallelogram.

Materials required :

- (i) White and coloured papers
- (ii) Pencil, eraser
- (iii) Geometrical instruments
- (iv) Sketch pens
- (v) Fevicol
- (vi) White chart paper





Notes

Preparation for the activity: -

- (i) Take a chart paper of size $15\text{cm} \times 10\text{cm}$
- (ii) On a white sheet of paper, draw a parallelogram ABCD, with dimensions $AB=4\text{cm}$, $BC=3\text{cm}$ and $\angle ADC = 75^\circ$
- (iii) Fold the parallelogram in such a way that CD falls on AD and press to get the crease BD. Draw a line along the crease.
- (iv) Paste the parallelogram ABCD on the piece of white chart paper with the help of fevicol.
- (v) Through A, draw a line AE parallel to BD to meet CB produced in E. Join DE.

Demonstration and Use :

1. Colour the two triangles BEF and ADF in blue and violet colours respectively.
 2. Make a replica of $\triangle ADF$ and super pose it on $\triangle BEF$ such that AD falls along BE and AF along BF.
 3. You will find that the two triangles cover each other completely they area equal in area.
 4.
$$\begin{aligned} \text{ar (Paralelogram ABCD)} &= \text{ar (Quad DCBF)} + \text{ar (} \triangle \text{ DAF)} \\ &= \text{ar (Quad. DCBF)} + \text{AR (} \triangle \text{ FBE)} \\ &= \text{ar (} \triangle \text{ DCE)} \end{aligned}$$
- \therefore Parallelogram ABCD and $\triangle DCE$ are equal in area.

ACTIVITY 30



Notes

Title: To find the incentre of different types of triangles

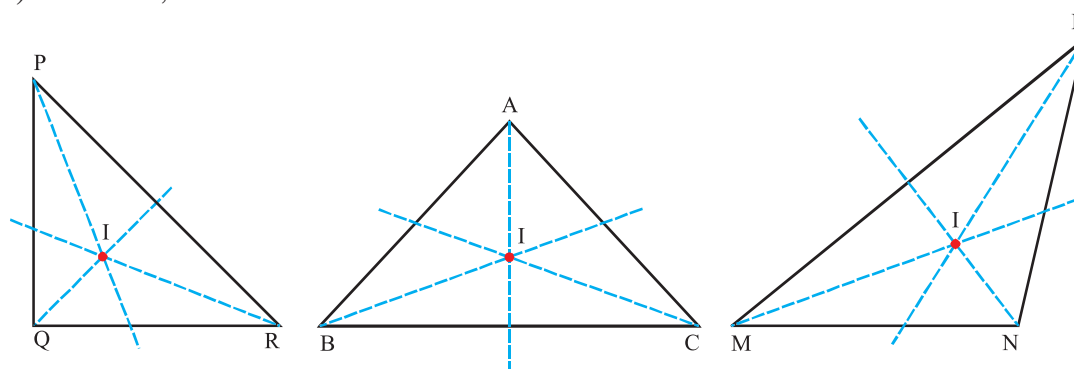
Expected Background Knowledge:

- (i) Different types of triangles
- (ii) Concurrent lines in a triangle

Objectives:- After performing the activity, the learner will be able to find the incentre of any given triangle.

Materials required :

- (i) White paper sheets
- (ii) Cutter
- (iii) Sketch pens
- (iv) Pencil, scale and eraser



Preparation for the activity:-

- (i) Take three sheets of paper each of size $8\text{cm} \times 10\text{cm}$ and draw a scalene triangle on one, a right triangle on the second and an obtuse triangle on the third sheet.
- (ii) Cut out the triangular portion from each of the sheets with the help of a cutter.
- (iii) Draw the angle bisectors of the angles of these triangles by paper folding, after making the respective creases.

Demonstration and Use :

You will find that the creases (the angle bisectors) are concurrent at a point. This point is called the incentre of the triangle.